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Spatial Thinking Development

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Spatial Thinking; Definition

The understanding of space, of spatial concepts, and of spatial phenomena, the knowledge of interrelations and interactions among them but also the relation and action between humans and the environment constitute a significant cognitive ability; **spatial thinking**.

Why Spatial Thinking?

- Ability with profound and rewarding effects on numerous aspects of everyday life and **STEAM** disciplines, also highly relevant to social sciences and humanities - from giving and following directions, interpreting maps and diagrams, to understanding correlation of social phenomena in space and time.

- Replaced, however, in education for a long period of time by other forms of thinking (verbal, metaphorical, hypothetical, and mathematical).

Spatial Thinking; The importance of scale

Which space?

Context (or Scale)	Cognition 'in' space (e.g., navigation in streets)	Cognition 'about' space		Cognition 'with' space (e.g., making concept maps)
		Geographic scale (e.g., studying maps)	Micro or astronomical scales (e.g., explaining structure of the atom or the solar system	
Learning to Think Spatially (2006)	←——	——————	——————	——→
Gersmehl & Gersmehl (2006, 2007) & Golledge et al. (2008)		←——→		

Scales of spatial thinking concepts used in the literature, from Lee and Bednarz, 2012

Spatial Thinking; The importance of scale

Montello's typology (1993)

- **Figural space**; perceived in all its properties, without requiring locomotion by humans
- **Vista space**; which includes the human body, relatively as large or larger than the human body and comprises a room or a town square
- **Environmental space**; larger than the human body, comprising semantic information, variable and hardly perceptible without moving into it, comprises of buildings, neighborhoods, cities and, finally,
- **Geographical space**; much larger of the human body and cannot be understood directly even by human movement. Conversely, it can be perceived through representations, such as maps or schematic models.

Spatial Thinking; Related Notions

- **Spatial sense** can be defined as an **intuition about shapes and the relationships among shapes**. Individuals with spatial sense have a feel for the geometric aspects of their surroundings and the shapes formed by objects in the environment (van de Walle, 2003).

- **Space perception** is defined as the perception of the properties and relationships of objects in space especially with respect to **direction, size, distance, and orientation** (Merriam - Webster Dictionary, 2014).

Spatial Thinking; Related Notions

- **Spatial ability** is the ability to:
  - understand and remember the **spatial relationships** among objects
  - manipulate **images in space**
  - **visualize** how separate parts of complex physical systems **interrelate** (The Johns Hopkins University Center for Talented Youth, 2013)

Spatial Thinking; Related Notions

- Spatial abilities include the following (Golledge, 1992):
  - **thinking geometrically**;
  - **imaging complex spatial relations at various scales**;
  - **recognizing spatial patterns** in distributions of functions, places and interactions at a variety of different scales;
  - **interpreting macro-spatial relations** such as star patterns;
  - **giving and comprehending directional and distance estimates** as required by navigation, or the path integration and short-cutting procedures used in way finding;
  - **understanding network structures** used in planning, design and engineering; and
  - **identifying key characteristics of location and association of phenomena in space**.

Spatial Thinking; Related Notions

- Spatial thinking includes **cognitive skills** related to:
  - **map reading and making,**
  - processes involving **representation, scale transformation, production and recall of symbolic (non-verbal) information, recognition and understanding of spatial projections, coordinate systems, synthesis of geometric configurations, formulation of verbal instructions,** and
  - **navigation and orientation based on observation and instruments handling**.
- Finally, the distinction between **knowledge of space** and **knowledge about spaces** is significant; *knowledge of space is phenomenal, knowledge about spaces is intellectual* (Eliot, 2000)
- Piaget also argues that **interaction in space**, not perception of space is a fundamental building block for the acquisition of **spatial knowledge** in (Golledge & Stimson, 1997, p. 191).

Axes of spatial thinking

- Spatial skills (cognitive scientists taxonomy):

1. Disembedding
2. Spatial visualization
3. Mental rotation
4. Spatial perception
5. Perspective taking

Axes of spatial thinking

Spatial Skills (geographic information scientists views):

- Navigation/ orientation
- Estimating distances and directions (angles)
- Map literacy
- Recognizing and understanding patterns in the 3d world
- Select an ideal location based on the given spatial features (siting)
- **Visualization**
- Understand and identify spatial correlations/ dependencies
- Categorize spatial entities/ geographic features and identify hierarchies if any
- Compare spatial entities and draw analogies among them
- Understand and identify models of spatial organization
- Identify/ determine connections/ relations among spatial entities and between humans and spatial entities
- Understanding scale in space and time
- Delineation of spatial regions/ zones based on given features/ properties

The formation of spatial perception and knowledge

Three approaches to spatial development

1. Piaget’s account
2. Nativist approaches
3. Vygotsky’s theory

Piagetian spatial development

Piaget's premise: infants born without knowledge of space or of the things in space.

A series of experiments by Piaget and his colleagues (1956, 1958, 1960) on how children perceive space, thereby forming spatial perception and knowledge:

- spatial awareness from infancy beginning from near/far objects,
- distinguish various simple relationships between objects, which are of topological nature,
- topological ideas are very general (proximity, order, separation and closure) and can be described through the features of various types of geometry.

Piaget's stages of spatial perception

- 4 stages to a child's development;
    1. the sensorimotor stage (0-2 years),
    2. the pre-operational stage (2-7 years),
    3. the concrete operational stage (7-12 years), and
    4. the formal operational stage (12-18 years).

Piaget’s stages of spatial perception

- During the formal operational stage:
  - "reasoning using a **hypothesis** and testing it in a certain way”; involves hypothetical situations and is often required in **science and mathematics**.
  - **abstract thought**, **metacognition** (thinking about thinking) and **problem-solving** are also abilities acquired during this stage.

Piaget’s stages of spatial perception

Spatial knowledge during the formal operational stage

- During adolescence (from the 12th to the 18th year), children are at the stage of formal reasoning; the perception of space is growing. Specifically, through perception, all elements in both the area surrounding the children and the global area (in its all dimensions) are interpreted.
- **Space** in the mind of the teenager receives a new dimension and from sensorimotor and practical, **becomes reasonable, uniform and clear**.
- Children become able to understand and handle abstract and general concepts and relations, or even mathematical, which are studied per se, as abstract and independent.

Piagetian spatial development

Where Piaget was wrong

1. Many spatial achievements are reached late in childhood.
2. Adults perform spatial tasks accurately.

Seem contradictory? Yes(!), true, nonetheless.

Nativism

Nativist premise: understanding of space innate in infants.

- Empirical evidence to support the view:
  1. Early ability to perform spatial analysis even by blind children (no visual input used)
  2. Understanding space is a (geometric) modular ability
  3. Maturation of the brain can only help aspects of spatial development that were not accounted for by innateness.
- Nativism suffers from lack of interest in environmental input and developmental change; its focus on the origins of cognitive competence have proven quite monolithic.

Vygotsky’s influence

Vygotskyan views of : 1) Guided participation, 2) Situation specificity and 3) Symbolic systems have influenced research in spatial competence.

1. Guided participation; children better understand the world through adults/older peers guidance.
2. Situation specificity; cognitive effort is adapted to and specific of particular situations.
3. Symbolic systems; dealing with maps or diagrams involves how humans interact with their cultural environment. No direct experience is need, such systems serve as **amplifiers of individual intelligence**.

Counterargument : emphasis on adult instruction and on cultural transmission, leaving aside individual cognitive efforts.

Development

Question of:

Age/ Maturity?

Gender?

Experience?

Society/ Culture?

Instruction?

Learning Styles / Modalities and Spatial Thinking

- The VARK model (Fleming and Mills, 1992)
  - Visual
  - Aural/ Auditory
  - Read/Write
  - Kinesthetic

Cross-cutting Concepts in Science Education

1. Patterns
2. Cause and effect
3. Scale, proportion, and quantity
4. Systems and system model
5. Energy and matter
6. Structure and function
7. Stability and change

(NRC, 2013)

Cross-cutting Concepts and Spatial Thinking

- Patterns in the distribution of the most populated cities worldwide are identifiable.
- Talking about natural hazards includes scale, proportion, or magnitude.
- Geographic realms are composed of different systems, with different structures and functions each, and are ruled by unique system models.
- At times geographic realms present a certain kind of stability, while most of the times they are in constant change and this can be the cause of different transformative effects on systems, structures, functions, quantities, etc.
- Events occurring in space, need energy in order to transpire that may lead to a change in matter.

Spatial thinking development and the curriculum

van Hiele's model of geometric thought development

A pedagogical framework for Geometry; the van Hiele Model of the Development of Geometric Thought [1959] (van Hiele-Geldof & van Hiele, 1984)

- Covers **geometric thought development from 5year-olds to adults**.
- Identifies **5 levels** of geometric understanding.

van Hiele's model of geometric thought development

1. Visualization; recognize and name shapes based on global, visual characteristics of the shape.
2. Analysis; able to consider all shapes within a class and recognize properties of classes of shapes.
3. Informal deduction; develop relationships between and among these properties.
4. Formal deduction; construct proofs; understand the interaction of necessary and sufficient conditions; make distinctions between a statement and its converse.
5. Rigor; understand non-Euclidean geometries and different systems can be compared; geometry is seen in the abstract.

van Hiele's model of geometric thought development

- The learner progresses sequentially from level 1 to 5, throughout the years of their academic endeavor.
- **Not** all learners make it to level 5, though because few of them are exposed to it through the formal curricula.
- This last level has received little attention and analysis by researchers, since the majority of high school geometry curricula end at level 4 (Crowley, 1987).

van Hiele’s model of geometric thought development

The properties of the model identified by van Hiele himself are:

- Sequential; the levels come one after the other; the learner should have mastered the previous level to proceed to the next.
- Advancement; progress form one level to another comes not from age rather from (successful) teaching methods.
- Intrinsic and extrinsic; inherent concepts of one level become study concepts of the next.
- Linguistics; students master concept names gradually; a square is also a rectangle and a parallelogram as the students move from one level to the next.
- Mismatch; occurs when teaching methods belong to superior level than of the student’s.

(Geo)spatial concepts and educational needs

Golledge et al. (2008a) and (2008b) have proposed a geospatial task ontology as an effort to match geospatial concepts with geographic educational needs

- The ontology consists of **45 geospatial concepts** organized in **five categories**, from basic concepts called primitives to complex ones, which are 4th order derivatives.

(Geo)spatial concepts and educational needs

- A child should first understand simple spatial concepts before proceeding to more complex ones, because complex concepts result from the combination of several simpler ones. ➡

- For every level of complexity several tasks have been proposed that would help children to better understand the concepts of each level

(Geo)spatial concepts and educational needs

- Some examples of tasks associated to concepts at different levels:
  - Primitives; **Location**: Recognizing that objects are found or located at specific places (e.g., home, school, shopping, gas station)
  - Simple concepts; **Edge/boundary**: Awareness of containment within a boundary (e.g., city; school yard; shopping mall)
  - Difficult concepts; **Center**: Determining (by estimation, measurement, or common acceptance) the middle of a spatial set (such as “the center of the city”)
  - Complicated concepts; **Connectivity**: Building or recognizing a static or dynamic area surrounding a node (e.g., newspaper circulation; marketplace)
  - Complex concepts; **Social Area**: Recognizing or constructing regions based on social characteristics of people (e.g., families versus singles)

(Geo)spatial concepts and educational needs

- Empirical evidence: studies and experiments indicate that the ontology can be used as a reference frame for the development of the Geography curriculum.

Identity
Location
Magnitude
Space-time

Tier	Geospatial concept	K	1	2	3	4	5	6	7	8	9	10	11	12
Primitive	Spatial Primitives	X	X	X	X	X	X	X	X	X	X	X	X	X
Simple	Relative Distance/Direction	X	X	X	X	X	X	X	X	X	X	X	X	X
	Shape		X	X	X	X	X	X	X	X	X	X	X	X
	Place-based Symbol		X	X	X	X	X	X	X	X	X	X	X	X
Difficult	Boundary			X	X	X	X	X	X	X	X	X	X	X
	Connection			X	X	X	X	X	X	X	X	X	X	X
	Distribution				X	X	X	X	X	X	X	X	X	X
	Pattern				X	X	X	X	X	X	X	X	X	X
	Reference Frame				X	X	X	X	X	X	X	X	X	X
	Coordinate/Grid				X	X	X	X	X	X	X	X	X	X
	Zone					X	X	X	X	X	X	X	X	X
Complicated	Map						X	X	X	X	X	X	X	X
	Legend						X	X	X	X	X	X	X	X
	Map Projection						X	X	X	X	X	X	X	X
	Slope/Gradient							X	X	X	X	X	X	X
	Scale							X	X	X	X	X	X	X
	Surface								X	X	X	X	X	X
	Hierarchy										X	X	X	X
	Overlay										X	X	X	X
Complex	Interpolation											X	X	X
	Global Warming												X	X
	Spatial Association												X	X

Connection of geospatial concepts to education levels from Golledge, Marsh, & Battersby, 2008a

(Geo)spatial concepts and educational needs

- Concepts of the same level of complexity in the Golledge ontology cannot be grasped by children at the same time.

Tier	Geospatial concept	Grade												
		K	1	2	3	4	5	6	7	8	9	10	11	12
Primitive	Spatial Primitives	X	X	X	X	X	X	X	X	X	X	X	X	X
Simple	Relative Distance/Direction	X	X	X	X	X	X	X	X	X	X	X	X	X
	Shape		X	X	X	X	X	X	X	X	X	X	X	X
	Place-based Symbol		X	X	X	X	X	X	X	X	X	X	X	X
Difficult	Boundary			X	X	X	X	X	X	X	X	X	X	X
	Connection			X	X	X	X	X	X	X	X	X	X	X
	Distribution				X	X	X	X	X	X	X	X	X	X
	Pattern				X	X	X	X	X	X	X	X	X	X
	Reference Frame				X	X	X	X	X	X	X	X	X	X
	Coordinate/Grid				X	X	X	X	X	X	X	X	X	X
	Zone					X	X	X	X	X	X	X	X	X
Complicated	Map						X	X	X	X	X	X	X	X
	Legend						X	X	X	X	X	X	X	X
	Map Projection						X	X	X	X	X	X	X	X
	Slope/Gradient							X	X	X	X	X	X	X
	Scale							X	X	X	X	X	X	X
	Surface								X	X	X	X	X	X
	Hierarchy										X	X	X	X
	Overlay										X	X	X	X
Complex	Interpolation											X	X	X
	Global Warming												X	X
	Spatial Association												X	X

(Geo)spatial concepts and educational needs

- As an overall conclusion, one could say that contrary to *simple* and *difficult* level concepts (1st and 2nd order derivatives) which become all best understood by the age of 10 and need less time to become so, *complicated* and *complex* level concepts (3rd and 4th order derivatives) need more time to become fully understood and they all do so from the age of 10.
- What is important to bear in mind is that spatial knowledge can be acquired by humans from the early stages of their education. In the beginning, the child is exposed to geospatial primitives and gradually progresses into more complex concepts along the way.

What is **lacking or impeding** STEM education and spatial thinking in the classrooms and beyond

Use of pedagogical approaches

Most used:

- Traditional direct instruction ⬇

Least used:

- **Peer teaching (Students are provided with opportunities to teach other students)** ⬆
- Personalized learning (Teaching and learning are tailored to meet students' individual interests and aspirations as well as their learning needs)
- Flipped classroom (Students gain the first exposure to new material outside of class, and then use classroom time to discuss, challenge and apply ideas or knowledge) ⬆

Use of different aspects of teaching and learning

Least used:

- Students give presentations to the whole class
- Students make decisions about how they learn
- Students conduct their own study and research activities
- Organize field trips/ visits to museums/ company visits to contextualize teaching content

Use of different types of learning resources

Least used:

- Web-based or computer-based simulations
- Online collaborative tools, e.g., Padlet, Mentimeter, Tricider, Kahoot
- STEM-specific software, e.g., Geogebra, Function Plotter
- Resources for personalized learning
- Data sets / Spreadsheets
- Resources for special needs learners

Teaching strategies for developing students' spatial thinking skills

- Focus on problems
- Teamwork
- 21st century skills, e.g. creativity, innovation, communication
- Integration of content, e.g. multi-/inter-disciplinary approach
- Design
- Hands-on
- Inquiry
- Student-centered
- Assessment

Tools to develop spatial thinking

Teaching tools for developing students' spatial thinking skills

- Navigation and location apps
- Online maps and other visualizations
- Satellite images
- GPS devices
- Hands-on educational and laboratory equipment
- Maps, graphs, diagrams, images
- Physical models
- Simulations
- Software (e.g. Geographic Information Systems)
- Measuring instruments, e.g. for surveying

Maps and other representations

- Representations serve as an effective reasoning tool and trigger complex reasoning processes and abilities (National Research Council, 2006).
- Representations, which include maps, models, diagrams, and graphs help in making the most abstract concepts understandable, and additionally helps grounding their communication skills (Mathewson, 1999).
- Science and technology are developed through exchange of information and data presented as diagrams, illustrations, maps, schematics, which summarize information and contribute to their understanding by the wider public (ibid).
- Symbolic representations of spatial location serve the transmission of information among people, ensuring the need to explore each site they visit. (Newcombe & Huttenlocher, 2000).
- Uttal (2000) found out that using maps and thinking about them can help children to understand abstract concepts of space and to gain systematic thinking about spatial relations with which they have not come into direct contact. In addition, the “exposure” to maps can help children to think the numerous spatial relationships that may exist among locations.

Maps and other representations

Representations are all around us:

- Letters of the alphabet
- The sound of the word “mother”
- A red traffic light
- Gestures in sign languages
- …

Visual literacy

- The ability to recognize and understand ideas conveyed through visible actions or images
- The ability to make meaning from anything we see
  - the ability to interpret, negotiate, and make meaning from information presented in the form of an image, extending the meaning of literacy, which commonly signifies interpretation of a written or printed text. Visual literacy is based on the idea that pictures can be "read" and that meaning can be through a process of reading
- The ability to evaluate, apply, or create conceptual visual representations

What is visual litracy?

Why visual literacy?

- Most of spatial thinking is visual or at least entails some kind of visualization (mental or physical)
- It is one if 21st century skills
- Most scientists are visually literate
- Science ideas are best communicated/explained through visuals
- Science ideas are best learnt through drawing (Picturing to Learn)
- It is a visual world!
  - Everything we see is an image
  - In 2013 young people were exposed to visuals 7:38 hours per day

Visual literacy in the classroom

- Means many things from film, dance, and mime through the use of diagrams, maps, and graphs to children's picture books
- It includes teaching students to critically analyze the images presented to them through any kind of media
- It also entails equipping students with the tools to **create** presentations that effectively communicate content
- Visual arts: painting, drawing, sculpture, design, photography, film, video, architecture
- Media: paintings, maps, diagrams, art work, photographs etc.

Visualizations and Representations

- The flavor network from “Flavor network and the principles of food pairing”, 2011, by Yong-Yeol Ahn et al.
- Islington has issues from “London: The Information Capital”, 2015, James Cheshire and Oliver Uberti
- Map of the Internet by Tim Berners Lee, 2007, and by Kevin Kelly, 2009
- The 2004 Indonesia Tsunami by Bruce Parkerand animation by Vasily V. Titov
- Minard’s Map, 1869,
- The true size of Africa, by Kai Krause
- Open Healthcare Access Map, the Heidelberg Institute for Geoinformation Technology

Examples of artwork…

- Balcony, 1945 by M.C. Escher
- Tingari Cycle, by Warlimpirrnga Tjapaltjarri,
- The Ambassadors, 1533 by Hans Holbein the Younger and Contemporary anamorphic art
- Stars, 1948 by M.C. Escher
- System III, by, M.C. Escher
- Composition II in Red, Blue, and Yellow, 1930, by P. Mondrian
- Number 14: Gray, 1948, by Jackson Pollock

and their STEM counterparts

- German States Population
- London Tube Map (Network)
- Cylindrical Projections and Distortions in maps (Cartography)
- Ginsenoside (Chemistry)
- Tessellation (Math) and Voronoi diagram and TIN (GIS),
- The golden ratio (Architecture)
- Self-similarity of Coastlines and fractals (Nature)

STEAM education (spatial concepts therein)

- Space distortion, emphasis, proportion, scale, detail
- Connectivity, proximity, network, movement
- Geometry distortion, transformation, error, precision, accuracy, mapmaking
- 3d geometry, 3d space
- Symmetry, grid, reference, tessellation, shape, movement, pattern
- Harmony(?), rhythm, unity, balance
- Fractal, pattern, scale, shape
- Representation

Proposals for activities in the Arts

- Learners map their neighbourhood, mark on the map the spaces intended for young people and make a relevant planning proposal
- Making beautiful/illustrative maps
- Reproducing a famous painting, paying attention to the location of people/objects and light source/ shadows
- Shaping spatial objects, drawing with different view angles (perspective taking)
- Engineering
- Architecture/ drawing figures using a computer
- Combining technical and creative subjects to fascinate learners, using divergent thinking to find as many solutions as possible to a problem, looking at reality from multiple points of view; teaching mathematics through music
- Making science videos, for instance, for the social media

Open educational resources/tools relevant to spatial and environmental awareness

- PhET Interactive Simulations for Science and Math
- Actionbound
- GAM
- GeoGebra Math Apps
- Google Earth/ Google Maps
- ArcGIS
- Kahoot!
- Miro
- Tess1.4
- LearningApps.org
- LEIFIphysik (GE)
- OpenStreetMap
- QGIS
- Socrative
- Xsection
- 3D printing models of artwork and anamorphic objects

Conclusions

- The geospatial domain presents an excellent opportunity towards achieving a meaningful connection between theoretical, higher-level concepts (e.g., geographical phenomena and processes) and tools of representation (e.g., maps and terrain models) and their application in everyday life such as locating one's home or following directions to an unknown place using their mobile phones or web-based applications (reasoning processes).

- Since geospatial thinking varies according to age, background knowledge, education, etc., a major challenge is to analyze the needs and characteristics of students and develop the appropriate knowledge components that will help them enhance their geospatial skills.

Conclusions

- No consistent and significant research on how children in adolescence develop spatial thinking, only that they have learnt from early ages to deploy strategies to perform spatial tasks.

- Geospatial concepts cross-cut the curriculum which make it more difficult to identify how spatial thinking notions are dealt with in formal education.

- To support spatial thinking in the classroom, it should be incorporated into the general education system including educational practices, curricula, teaching support materials, and assessments.

Referenced Bibliography

- Crowley, M. L. (1987). The van Hiele Model of the Development of Geometric Thought. In L. M. M. (Ed.), *Learning and Teaching Geometry, K-12, 1987 Yearbook of the National Council of Teachers* (pp. 1-16). Reston, Va: National Council of Teachers of Mathematics.
- Eliot, J. (2000). *The nature and measurement of spatial intelligence.* College Park, MD: Institute for Child Study, College of Education, University of Maryland.
- Fleming, N.D. & Mills, C. (1992). Not Another Inventory, Rather a Catalyst for Reflection. To Improve the Academy, 11, 137-155.
- Golledge, R. G. (1992). Do People Understand Spatial Concepts: The Case of First-Order Primitives. *Proceedings of the International Conference GIS - From Space to Territory: Theories and Methods of Spatio-Temporal Reasoning in Geographic Space* (pp. 1-21). Springer-Verlag.
- Golledge, R. G., Marsh, M., & Battersby, S. (2008a). A Conceptual Framework for Facilitating Geospatial Thinking. *Annals of the Association of American Geographers, 98*(2), 285-308.
- Golledge, R. G., Marsh, M., & Battersby, S. (2008b). Matching Geospatial Concepts with Geographic Educational Needs. *Geographic Research, 46*(1), 85-98.
- Golledge, R. G., & Stimson, R. J. (1997). *Spatial Behavior: A Geographic Perspective.* New York: The Guilford Press.
- Lee, J., & Bednarz, R. (2012). Components of spatial thinking: Evidence from a spatial thinking ability test. *Journal of Geography*, 111(1), 15-26.
- Mathewson, J. H. (1999). Visual-spatial thinking: An aspect of science overlooked by educators. *Science Education, 83*(1), 33–54.

Referenced Bibliography

- Montello, D. R. (1993). Scale and multiple psychologies of space. In A. U. Frank, & I. Campari (Ed.), *Spatial information theory: A theoretical basis for GIS Proceedings of COSIT '93. 716*, pp. 312-321. Berlin: Springer-Verlag, Lecture Notes in Computer Science.
- National Research Council. (2006). *Learning to Think Spatially: GIS as a Support System in the K-12 Curriculum.* Washington, DC: The National Academies Press.
- National Research Council. (2013). Next Generation Science Standards: For States, By States. Washington, DC: The National Academies Press. Newcombe, N. S., & Huttenlocher, J. (2000). *Making Space.* Cambridge MA: MIT Press.
- Piaget, J., & Inhelder, B. (1956). *The Child's Conception of Space.* London: Routledge.
- Piaget, J., & Inhelder, B. (1958). *The Growth of Logical Thinking from Childhood to Adolescence.* New York: BasicBooks.
- Piaget, J., Inhelder, B., & Szeminska, A. (1960). *The child's conception of geometry.* London: Routledge and Kegan.
- The Johns Hopkins University Center for Talented Youth. (2013). What is spatial ability? Retrieved from http://web.jhu.edu/cty/STBguide.pdf
- Uttal, D. H. (2000). Seeing the big picture: Map use and the development of spatial cognition. *Developmental Science, 3*(3), 247-264.
- van de Walle, J. A. (2003). Geometric Thinking and Geometric Concepts. In J. A. van de Walle, *Elementary and Middle School Mathematics: Teaching Developmentally* (5th ed., pp. 407-453). Boston: Pearson.
- van Hiele-Geldof, D., & van Hiele, P. (1984). *English Translation of Selected Writings of Dina van Hiele-Geldof and Pierre M. van Hiele.* (F. D., G. D., & T. R., Eds.) Brooklyn, NY: City University of New York. Brooklyn College. School of Education.