GOSTEAM Hands-on Activity Template *(Classroom-Formal)*

Title:

Functions in two variables

Short Description (Max 500 words):

Students analyze und illustrate functions in two variables. So, the enhance their knowledge and skills about functions in one variable.

Keywords (Up to 5):

Functions, variables, mathematics, GeoGebra

Information about the Implementation

Age and language of the students: 9-12 12-15 15-18 18+

Language: Age: ☐ 9-12 ☐ 12-15 ■ 15-18 ☐ 18+

Number of Lessons – Duration (per lesson):

Number of Lessons: 1-2 Duration per Lesson: full time

Subjects:

For which subject(s) the activity is usable, is it an interdisciplinary activity?

Science ☐

Physics ☐ Chemistry ☐ Biology ☐ Geosciences ☐ Environmental ☐ Other ☐

Technology ☐

Engineering ☐

Arts ☐

Mathematics ■

Information about the Scenario

Curriculum and country:

Link of the current activity to the curriculum:

Country:	Austria	Class:	AHS	Grade:	10
Topic:	real functions (analyzing and illustrating)				

Objectives (Max 100 words):

Description of the learning objectives

Students analyze und illustrate functions in two variables. So, the enhance their knowledge and skills about functions in one variable.

Materials (Max 100 words):

Which resources and materials (software, hardware) are needed?

GeoGebra 6 (language English is available in the settings); Knowledge about real functions in one variable (polynomial, sin, cos) and if required conic sections.

Spatial concepts, skills and abilities:

Which spatial concepts and skills are covered by the activity?

Spatial concepts:

Primitives: Identity/Name ☐ Location ☐ Space/Time ■

Simple: Distance ☐ Direction ☐ Connectivity ☐ Movement ☐ Boundary ■ Shape/Area ■ Adjacency ☐

Difficult: Overlay ☐ Buffer ☐ Topology ■ Coordinate ■ Map ☐ Scale ☐ Shortest Path ☐ Navigation ☐ Surface ■ Slope/Gradient ■ Aspect ☐ Contour ■

Complex: Interpolation ☐ Map Projection ■ Spatial Dependency ☐

Other: [ ]

Spatial skills:

- [x] Map literacy
- [ ] Navigation/orientation
- [ ] Estimating distances and directions
- [x] Recognizing and understanding patterns/Understand and identify models of spatial organization
- [ ] Select an ideal location based on the given spatial features
- [x] Visualization
- [ ] Understand and identify spatial correlations/ dependencies
- [x] Categorize spatial entities/ geographic features and identify hierarchies
- [x] Compare spatial entities and draw analogies among them
- [ ] Identify/determine connections/relations
- [ ] Understanding scale in space and time
- [ ] Delineation of spatial regions/ zones based on given features/ properties

Short Description

Navigation/orientation: Finding one's way in unfamiliar environments, interpreting and giving walking and driving directions.
Estimating distances and directions: Measure paths, weighted distances, angles.
Map literacy: Using, interpreting/understanding, learning from, and communicating acquired spatial knowledge from maps, comprehension of geographic features represented as points, lines, or polygons.
Recognizing and understanding patterns/Understand and identify models of spatial organization. Delineation of spatial regions/zones based on given features/properties: Regionalization processes, pattern recognition and clustering identification in the 2d and/or the 3d world.
Select an ideal location based on the given spatial features: Single or multi-criteria siting and optimal areas identification.
Visualization: Visualizing spatial entities from written/oral verbal descriptions, from their 2d or graphical representations or through mental transformations; such as axis rotation or perspective taking.
Understand and identify spatial correlations/ dependencies: The ability to realize, identify and explain patterns, clusters and relevant spatial dependencies.
Categorize spatial entities/geographic features and identify hierarchies: Identify the hierarchical form of data and gradients between spatial entities.
Compare spatial entities and draw analogies among them: Calculate and compare different geometric objects' shapes, area and, boundaries.
Identify/determine connections/relations: The ability to identify links and common characteristics among spatial entities and between humans and spatial entities.
Understanding scale in space and time: The understanding of changes/transitions through space and time for different spatio-temporal scales.

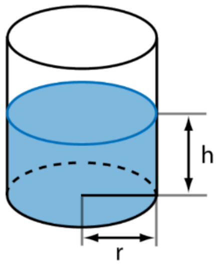
Geospatial concepts and spatial abilities documentation (see Section 3.2):
http://www.gosteam.eu/wp-content/uploads/2021/05/GOSTEAM_IO1_A1_final.pdf

Description of the activity in detail

Classroom & Online activities

Engage

Imagine a glass, which has the form of a cylinder. We fill some water in it and want to describe the volume V of the water, which forms a cylinder with height h and radius r.



Solution: $V(r,h) = r^2 \cdot \pi \cdot h$

Explain

The volume V is a function in two variables r and h. We define:

A function $f: A \to \mathbb{R}$ with $A \subseteq \mathbb{R}^2$ is called a **real function in two variables**. The function assigns every pair $(x,y) \in \mathbb{R}^2$ a real number.

In the real world, the function often describes a situation, where x and y online can be positive. Then, we write $(x,y) \in (\mathbb{R}^+)^2$.

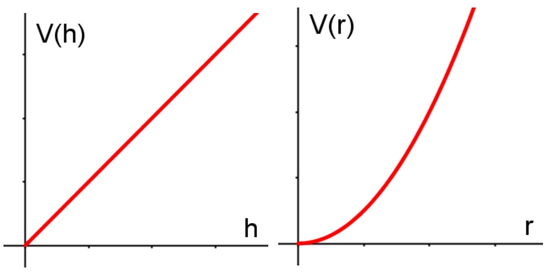
We can analyze und illustrate the function V, when we hold one variable and let the other one change. In this case, we get the graph of the function as a line in the x-$f(x)$-diagram.

$$V(h) = r^2 \cdot \pi \cdot h \qquad V(r) = r^2 \cdot \pi \cdot h$$

The function $V: h \longmapsto V(h)$ is linear and its graph is a line.

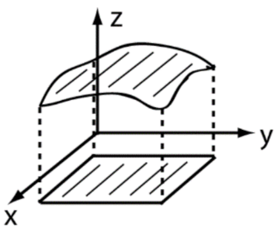
The function $V: r \longmapsto V(r)$ is quadratic and is graph is a parable.

It is possible to illustrate the graph of $V(r,h)$ at once? What would it look like?



Solution: The graph of $V(r,h)$ is an area in the 3-dimensional coordinate system.

A **function f in two variables x, y** can be illustrated as a **area in a 3-dimensional coordinate system**. Die z-axis gives the value of $f(x,y)$.

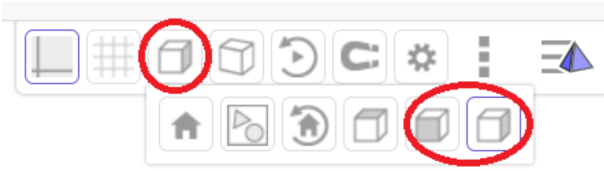


Explore

It is difficult to draw the graph of a function $f(x,y)$. Therefore, we use GeoGebra.

Technology information: Open GeoGebra 6. Click on the settings and choose *Perspectives* and then *3D Graphics*. On the left side, there is the algebra window, where we can type in any function. On the right side is the 3D graphic window, where the graph of the function will be shown.

Task: Type in some of the following functions in two variables and analyze them. Why does the graph look like this? Hint: Look at the functions $f_x(x,k)$, where y is constant and just x is changing and $f_y(k,y)$, where x is constant und just y is changing. You can look at this functions, when you click in the 3D Graphic window on the cubic.



(1) $f(x,y) = \frac{1}{2} \cdot (x^2 + y^2)$

(2) $f(x,y\,) = 0.1 \cdot (x^2 - y^2)$

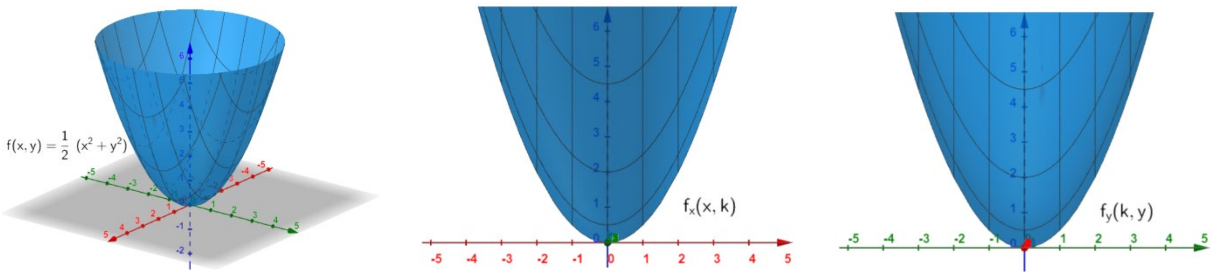
(3) $f(x,y\,) = \frac{1}{2}x^2 - y$

(4) $f(x,y\,) = \sin x \cdot \cos y$

(5) $f(x,y\,) = \sin(x + y)$

Solutions:

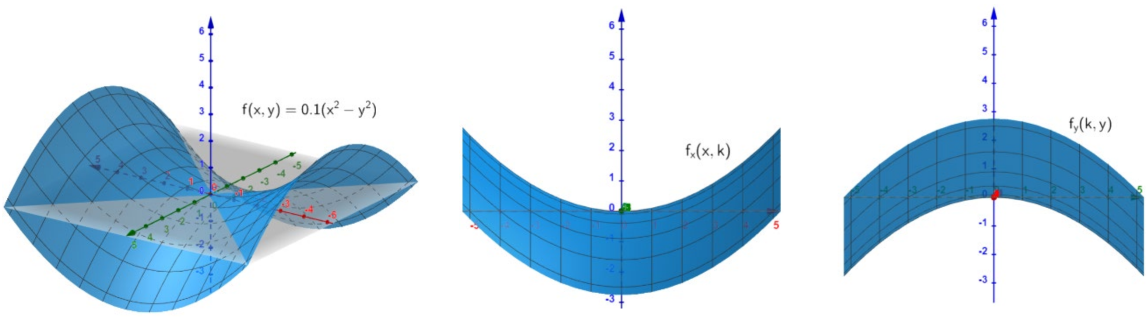
(1)



$f_x(x,k) = \frac{1}{2} \cdot (x^2 + k^2)$ … parables rising up, which are stretched by $\frac{1}{2}$ und shifted by $\frac{1}{2}k^2$ above the x-axis.

$f_y(k,y) = \frac{1}{2} \cdot (k^2 + y^2)$ … parables rising up, which are stretched by $\frac{1}{2}$ und shifted by $\frac{1}{2}k^2$ above the y-axis.

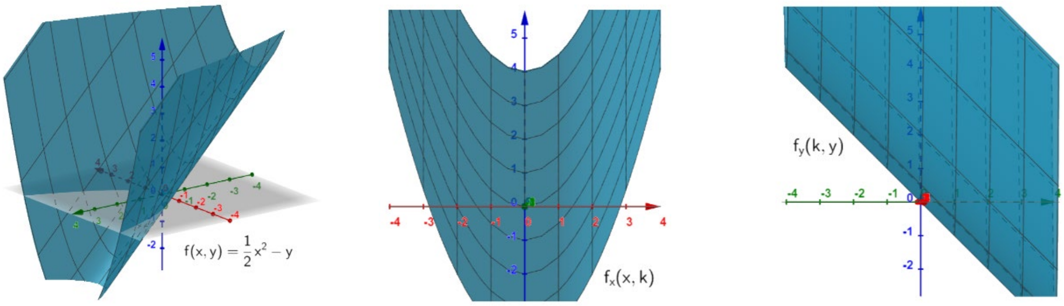
(2)



$f_x(x,k) = 0{,}1(x^2 - k^2)$ ... parables rising up, which are stretched by 0.1 und shifted by $0.1k^2$ underneath the x-axis.

$f_y(k,y) = 0{,}1(k^2 - y^2)$ ... parables rising down, which are stretched by 0.1 und shifted by $0.1k^2$ above the y-axis.

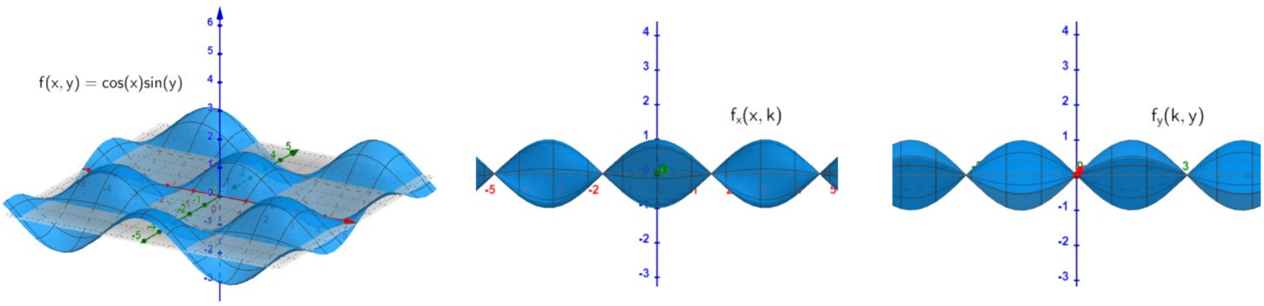
(3)



$f_x(x,k) = \frac{1}{2}x^2 - k$ ... parables rising up, which are stretched by $\frac{1}{2}$ and shifted by k above $(k < 0)$ or underneath $(k > 0)$ the x-axis.

$f_y(k,y) = \frac{1}{2}k^2 - y$ ... lines with decline -1 and cross the z-axis by $\frac{1}{2}k^2$.

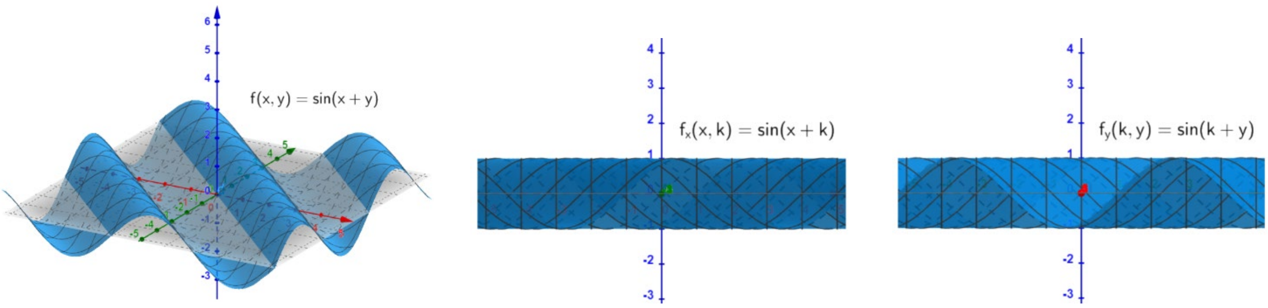
(4)



$f_x(x,k) = \cos x \sin k$ ... cos-functions, which are multiplied with $\sin k \in [-1;1]$

$f_y(k,y) = \cos k \sin y$ ... sin-functions, which are multiplied with $\cos k \in [-1;1]$

(5)



$f_x(x, k) = \sin(x + k)$ ... sin-functions, which are shifted by k to the left ($k > 0$) or to the right ($k < 0$)

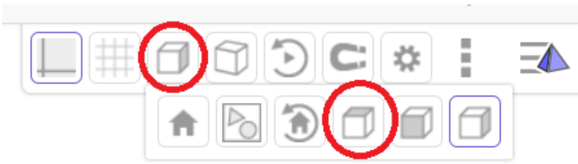
$f_y(k, y) = \sin(k + y)$ ... sin-functions, which are shifted by k to the left ($k > 0$) or to the right ($k < 0$)

Elaboration

An important term is the level line. We can find such level lines e. g. by maps, which contains mountains. The level line gives all positions with the same height above sea level.

Let $f: A \to \mathbb{R}$ with $A \subseteq \mathbb{R}^2$ be a function in two Variables x, y and $c \in \mathbb{R}$. The **level line of the number c** is the **amount of all points of A with $f(x, y) = c$**. This means, that all points of the level line have the same value of the function f.

We get the level line, when we cross the graph of f with a plain parallel to the x-y-plain and crossing the z-axis in the height c. In Geogebra we can choose one of the cubics to look from above.



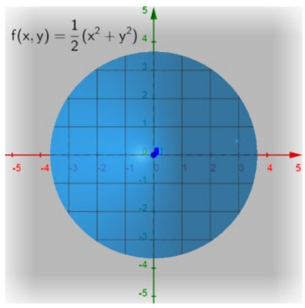
Task: Find the level lines of the previous functions (1), (2) and (3) and describe, why they look like this.

Solution

(1)

$\frac{1}{2} \cdot (x^2 + y^2) = c \quad \Leftrightarrow \quad x^2 + y^2 = 2c$

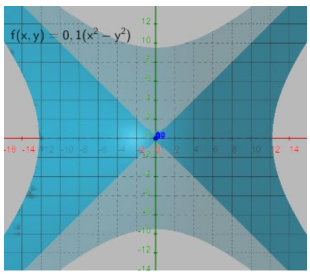
- For $c > 0$ ... circles with radius $\sqrt{2c}$
- For $c = 0$ ... Origin O
- For $c < 0$ ... no level line possible



(2)

$0{,}1\,(x^2 - y^2) = c \quad \Leftrightarrow \quad x^2 - y^2 = 10c$

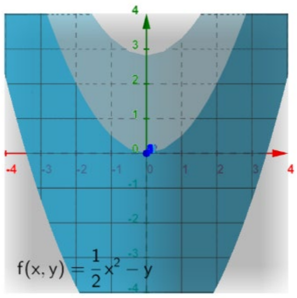
- For $c \neq 0$ ... hyperbolas
- For $c = 0$ ... two lines $y = x$ and $y = -x$



(3)

$\frac{1}{2}x^2 - y = c \quad \Leftrightarrow \quad y = \frac{1}{2}x^2 - c$

- Parables, which are rising up and shifted by c above ($c < 0$) or underneath ($c > 0$) the x-axis.



Sustainable contact:

Stephan Wastyn

stephan.wastyn@bildung.gv.at

References (if any):

Assessment (if any):