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GOSTEAM Hands-on Activity Template *(Inquiry-based)*

Title:

Optimal route finding and Least Cost Path concepts

Short Description (Max 500 words):

Route finding is one of the most common problems in spatial modelling and geosciences overall. Optimal route planning, distribution networks and navigation are some aspects that are covered in our daily life based on optimal pathfinding algorithms. In this scenario, students will explore how some common spatial routines work, in terms of the multi-objectivity and different distance metrics (Euclidean and 3D distance), some important spatial attributes (i.e., neighborhood, adjacency), as also, some simple spatial modelling schemes to delineate optimal paths. Consequently, this scenario will enforce our knowledge to identify whether our GPS is lying or not (?).

Keywords (Up to 5):

Least Cost Paths (LCP), Network, Spatial Data, Optimization

Information about the Implementation

Age and language of the students:

	9-12	12-15	15-18	18+
Language: Greek — Age:	☐	☐	☑	☐

Number of Lessons – Duration (per lesson):

Number of Lessons: 3 Duration per Lesson: 45’

Subjects:

For which subject(s) the activity is usable, is it an interdisciplinary activity?

- Science ☑
  - Physics ☐ Chemistry ☐ Biology ☐ Geosciences ☑ Environmental ☐ Other ☐
- Technology ☑
- Engineering ☑
- Arts ☐
- Mathematics ☑

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Information about the Scenario

Curriculum and country:

Link of the current activity to the curriculum:

Country: Greece | Class: Upper Secondary | Grade: 9 - 12

Topic: Mathematics, Applied Geometry, Routeing problems

Objectives (Max 100 words):

Description of the learning objectives

- Students will learn how the GPS navigation system works and overall, how pathfinding and routing problems are solved based on network (graph) structures.
- Students will be able to discuss, compare, and negotiate methods used, results obtained, and explanations among groups conducting the same investigation.
- The student will represent their ideas and solutions within text in the form of a report by incorporating graphs and maps.
- Utilize tools geographers use to study the world for solving complex engineering and geometry-based problems.

Materials (Max 100 words):

Which resources and materials (software, hardware) are needed?

Material: Search Engines, Spatial Data, Printed maps and Online Support Material

Software: QGIS platform

Spatial concepts, skills, and abilities:

Which spatial concepts and skills are covered by the activity?

Spatial concepts:

Primitives: Identity/Name ☐ Location ☑ Space/Time ☐

Simple: Distance ☑ Direction ☑ Connectivity ☐ Movement ☐

Boundary ☐ Shape/Area ☐ Adjacency ☑

Difficult: Overlay ☐ Buffer ☐ Topology ☐ Coordinate ☐

Map ☐ Scale ☐ Shortest Path ☑ Navigation ☑

Surface ☐ Slope/Gradient ☐ Aspect ☐ Contour ☐

Complex: Interpolation ☐ Map Projection ☑ Spatial Dependency ☐

Other:

Spatial skills:

- ☐ Map literacy
- ☑ Navigation/orientation
- ☑ Estimating distances and directions
- ☐ Recognizing and understanding patterns/Understand and identify models of spatial organization
- ☐ Select an ideal location based on the given spatial features
- ☑ Visualization
- ☐ Understand and identify spatial correlations/ dependencies
- ☐ Categorize spatial entities/ geographic features and identify hierarchies
- ☐ Compare spatial entities and draw analogies among them
- ☑ Identify/determine connections/relations
- ☐ Understanding scale in space and time
- ☐ Delineation of spatial regions/ zones based on given features/ properties

Short Description

Navigation/orientation: Finding one's way in unfamiliar environments, interpreting and giving walking and driving directions.
Estimating distances and directions: Measure paths, weighted distances, angles.
Map literacy: Using, interpreting/understanding, learning from, and communicating acquired spatial knowledge from maps, comprehension of geographic features represented as points, lines, or polygons.
Recognizing and understanding patterns/Understand and identify models of spatial organization. Delineation of spatial regions/zones based on given features/properties: Regionalization processes, pattern recognition and clustering identification in the 2d and/or the 3d world.
Select an ideal location based on the given spatial features: Single or multi-criteria siting and optimal areas identification.
Visualization: Visualizing spatial entities from written/oral verbal descriptions, from their 2d or graphical representations or through mental transformations; such as axis rotation or perspective taking.
Understand and identify spatial correlations/ dependencies: The ability to realize, identify and explain patterns, clusters and relevant spatial dependencies.
Categorize spatial entities/geographic features and identify hierarchies: Identify the hierarchical form of data and gradients between spatial entities.
Compare spatial entities and draw analogies among them: Calculate and compare different geometric objects' shapes, area and boundaries.
Identify/determine connections/relations: The ability to identify links and common characteristics among spatial entities and between humans and spatial entities.
Understanding scale in space and time: The understanding of changes/transitions through space and time for different spatio-temporal scales.

Geospatial concepts and spatial abilities documentation (see Section 3.2):
http://www.gosteam.eu/wp-content/uploads/2021/05/GOSTEAM_IO1_A1_final.pdf

Description of the activity in detail

Question Eliciting Activities

Provoke curiosity

Describe ways and materials that teachers will present to their students to attract their attention to the topic investigated.

Usually, the most effective way to provoke students' curiosity is by presenting an exciting video or a series of photos or even using some interesting games or hands-on activities!

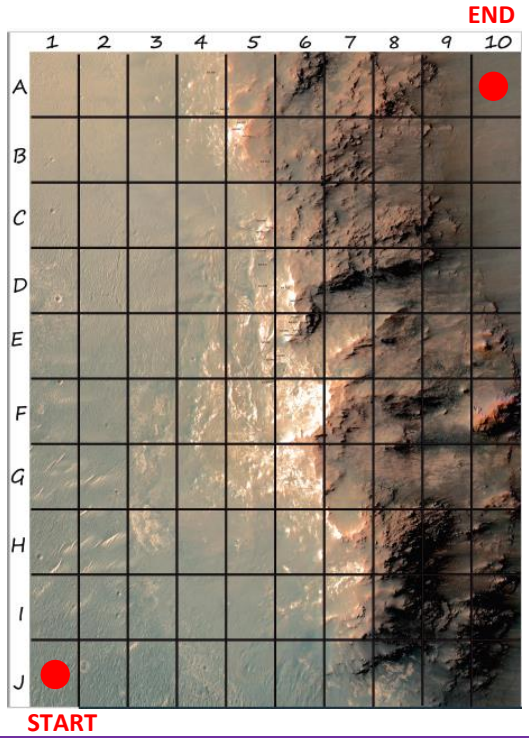
Before the activity begins you can print the map on the right for each student and ask from them to delineate an optimal path from the starting to the ending point.

They can move through each cell from each cell's center to another from and to all directions!

When the students finish their drawings and write their answers on the backside of the map ask them:

Which criteria you used to select your path?

Or why you selected this specific path?



Propose preliminary explanations or hypotheses

Formulate the scientifically oriented questions that teachers will present to the students to trigger their engagement in thinking about the topic investigated based on their existing knowledge. Make these questions digitally available and easily usable, e.g., by integrating them in the materials described in the previous step.

It is best to ask these questions in the context of a relative discussion.

To answer to the above-mentioned questions, we have to consider what type of problems the GPS solves? What do we want from our GPS?

To delineate the shortest path! Or maybe better, the fastest path!

Therefore, what is the fastest path to go to the school canteen?

Or maybe the shortest? (in terms of the distance covered) Or the safest?

In our daily activities when do we use or find pathfinding problems?

Active Investigation

Plan and conduct simple investigation

Offer instructions about the activities the students will need to perform and what kind of materials they may need. Describe ways in order to facilitate the students to focus on evidence.

This is the phase in which students are being prepared for the subsequent phase of evidence gathering during observation.

First of all, students have to realize the actual scientific problem and topic, hence, what a Least Cost Path (LCP) algorithm is!

Useful resources on the topic:

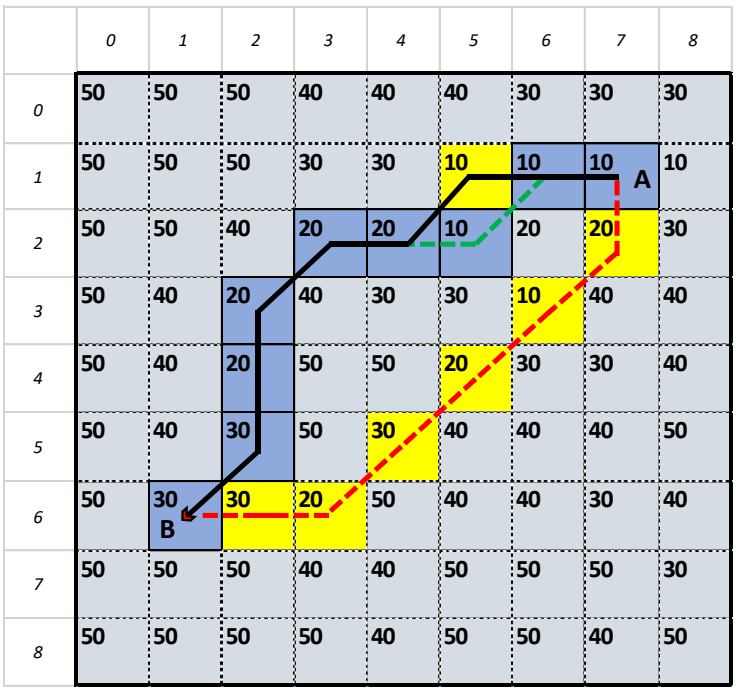
https://gisgeography.com/least-cost-path-analysis/ **(Examples of LCP)**

Route planning, pipelines and cables delineation, engineering applications, robotics, archaeology, animals' corridors, hiking trail planning and many more!

http://www.geography.hunter.cuny.edu/~jochen/gtech361/lectures/lecture11/concepts/Least-cost%20path%20analysis.htm **(Least Cost Path concept explanation)**

https://blog.routific.com/route-optimization-google-maps-313a45e13d27 **(Route optimization planning and GPS)**

In the following image you can see an indicative example of LCP implementation:



Path	x	y	z	ED	DH	RD
A	7	1	10			
	6	1	10	1	0	1
	5	2	10	1,414	0	1,414
	4	2	20	1	10	10,05
↓	3	2	20	1	0	1
	2	3	20	1,414	0	1,414
	2	4	20	1	0	1
	2	5	30	1	10	10,05
B	1	6	30	1,414	0	1,414
				9,243		27,34

Path	x	y	z	ED	DH	RD
A	7	1	10			
	6	1	10	1	0	1
	5	1	10	1	0	1
	4	2	20	1,414	10	10,1
↓	3	2	20	1	0	1
	2	3	20	1,414	0	1,414
	2	4	20	1	0	1
	2	5	30	1	10	10,05
B	1	6	30	1,414	0	1,414
				9,243		26,98

Path	x	y	z	ED	DH	RD
A	7	1	10			
	7	2	20	1	10	10,05
	6	3	10	1,414	-10	10,1
	5	4	20	1,414	10	10,1
↓	4	5	30	1,414	10	10,1
	3	6	20	1,414	-10	10,1
	2	6	30	1	10	10,05
	1	6	30	1	0	1
B				8,657		61,5

Figure 1: LCP considering 3D distance and slopes

Using this specific example and considering that the moving distance from one cell to another in the cross directions (up, down, left, right) is 1, estimate the diagonal distance along with the 3D distances from A to B (black, green and red paths).

ED: Euclidean distance, **DH:** Height difference, **RD:** Real distance on the ground

Which is the shortest distance considering only the 2-dimensional space (from one cell to another)?

To the contrary, if we also consider slopes (3-dimensional space), which is the shortest path and why?

Compare the results!

Creation

Gather evidence from observation

The selected resource (e.g., a simulation, an experiment, an animation, a graph, or other exhibit of similar nature) must provide students with an opportunity to collect evidence addressing the scientific questions presented in previous stages through direct or indirect observation.

It is recommended to introduce group work at this stage. Teachers can divide students in groups, each of which will be facilitated by the teacher to formulate and to evaluate explanations to the scientific questions based on the collected evidence.

Introduce all materials needed during the Activity along with short explanations:

https://www.youtube.com/watch?v=8oEnJvLzDnQ (What is QGIS?)

In order to better understand what type of data we will use and how we "translate" or model the Earth surface and processes, we have to discuss about spatial data structures (type of data and how spatial information is stored, data types etc.)

https://www.youtube.com/watch?v=HwVFvHwuYJo (Vector and Raster data)

QGIS Platform Download (version 3.14.0):

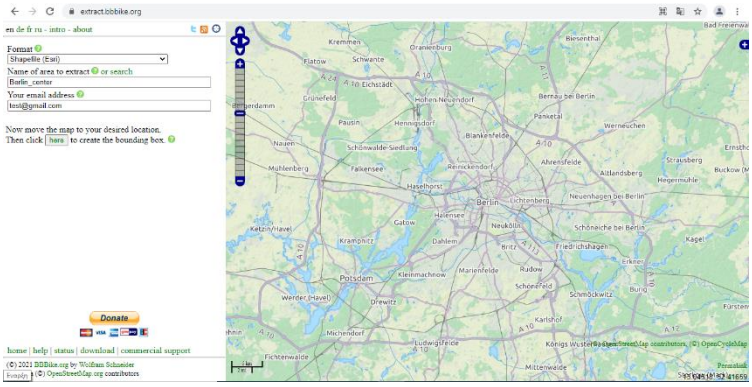
https://qgis.org/downloads/

Download vectorized data for any place you want worldwide:

https://extract.bbbike.org/

Before the beginning of the activity, students are separated to groups of 2. One of the group members is responsible for the tasks' implementation, data acquisition and the appropriate steps to be followed during the activity. The second member is responsible for the results validation and communication, including the final report and the maps and results preparation and the results reporting.

Step 1: Download spatial data in Shapefile format (vector data)



Visit:

https://extract.bbbike.org/

Figure 2: Bbike interface with the type of data an the area you want to search on the left and the global map on the right

1. Navigate to the map by zooming in and out and dragging the map to the desire area you want to acquire data.
2. Then select click here on the left of your screen to create the bounding box (See Figure 2).
3. Draw a rectangle, fill your email address, and select **Extract**. The download link will be sent to your email account!

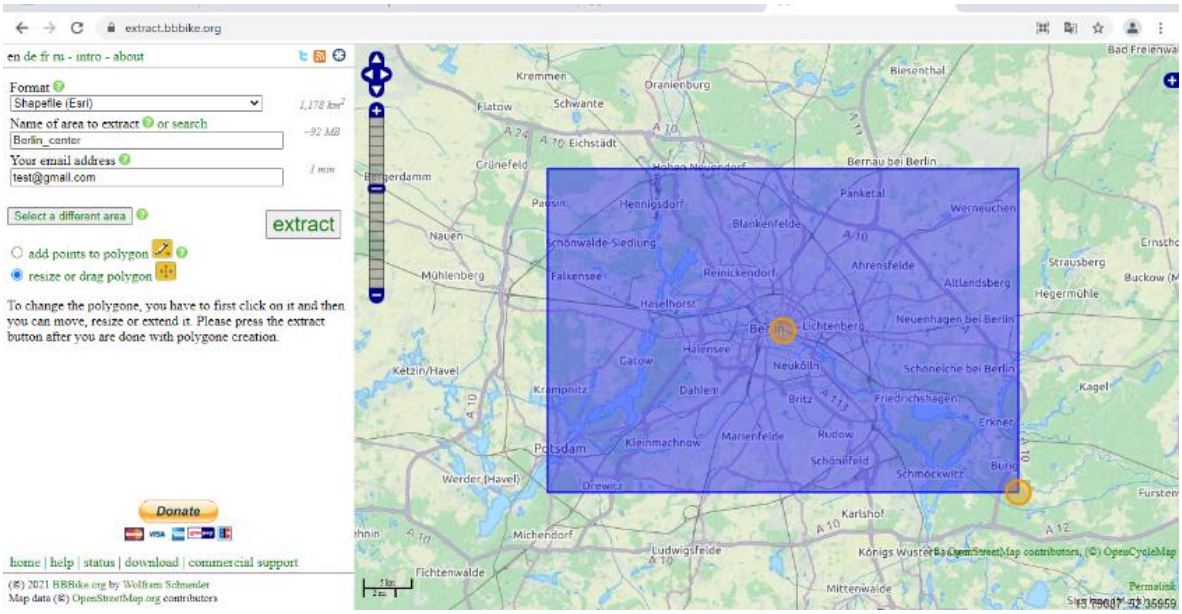


Figure 3: Drawing a rectangle to select your study area

Step 2: Download QGIS Interface (free online platform)

Visit the following link and download **QGIS 3.14** version depending on your operating system (OS).

After installation finishes, from the Start menu on your desktop, navigate to the QGIS folder and open ***QGIS Desktop 3.14.0 with GRASS 7.8.3*** and you will see the main window of QGIS platform as illustrated in **Figure 4**.

In **Figure 4** you can see some key functionalities and menus of the QGIS platform. The login of this activity is to load the vector data we have already downloaded on Step 1 and process these data in order to extract some minimum cost paths, for example, from our home to school!

To succeed that you have to: i) Load the data and ii) select the appropriate tools (algorithms) for delineating such paths upon different preferences.

1. *Navigate to your Downloads folder and "unzip" or "unrar" your compressed data.*
2. *On QGIS use the Main toolbar (top of the screen) -> Layers -> Vector Layer -> Navigate to your folder and select* ***roads.shp***
3. *Change layout colors using Layer Properties (double-click on the .shp file you've loaded)* ***-> Symbology -> Single line -> color****. You can also change the line width or style (Stroke width)*

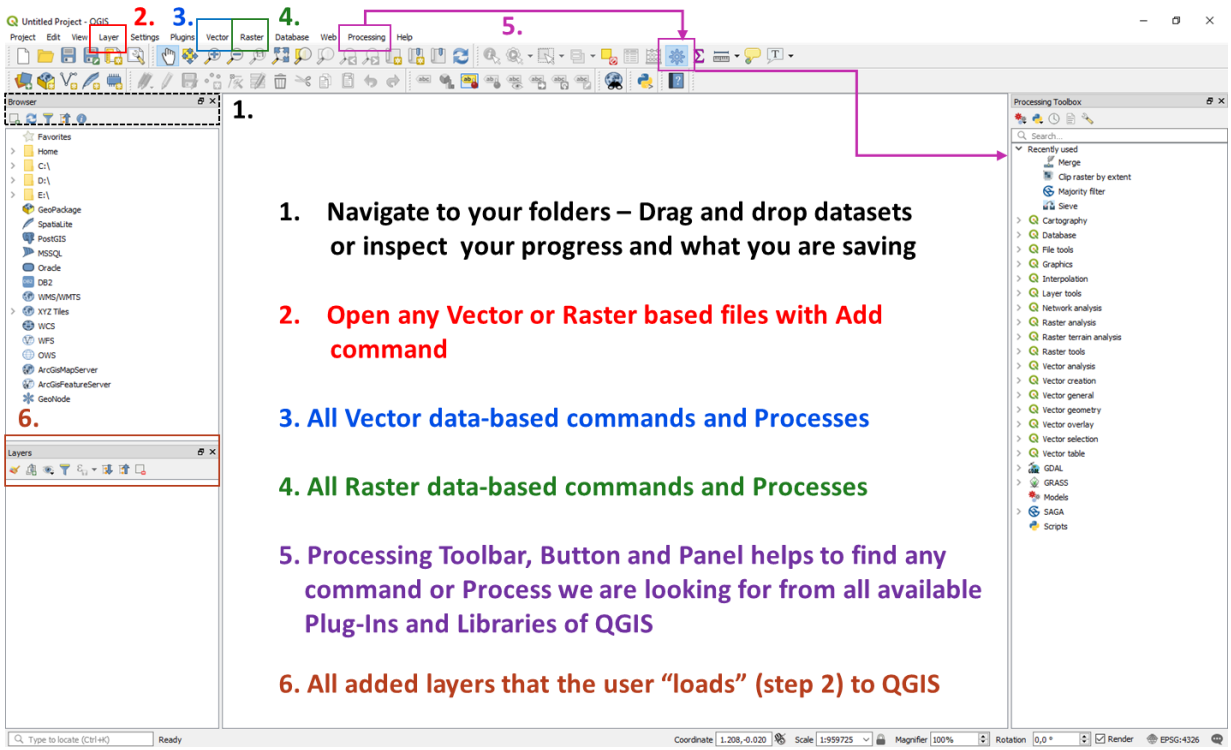


Figure 4: QGIS interface and key functionalities

After you load your data and adjust you line colors (i.e to grey) you will see the results as illustrated in **Figure 5**. You can also zoom-in or zoom-out to your area.

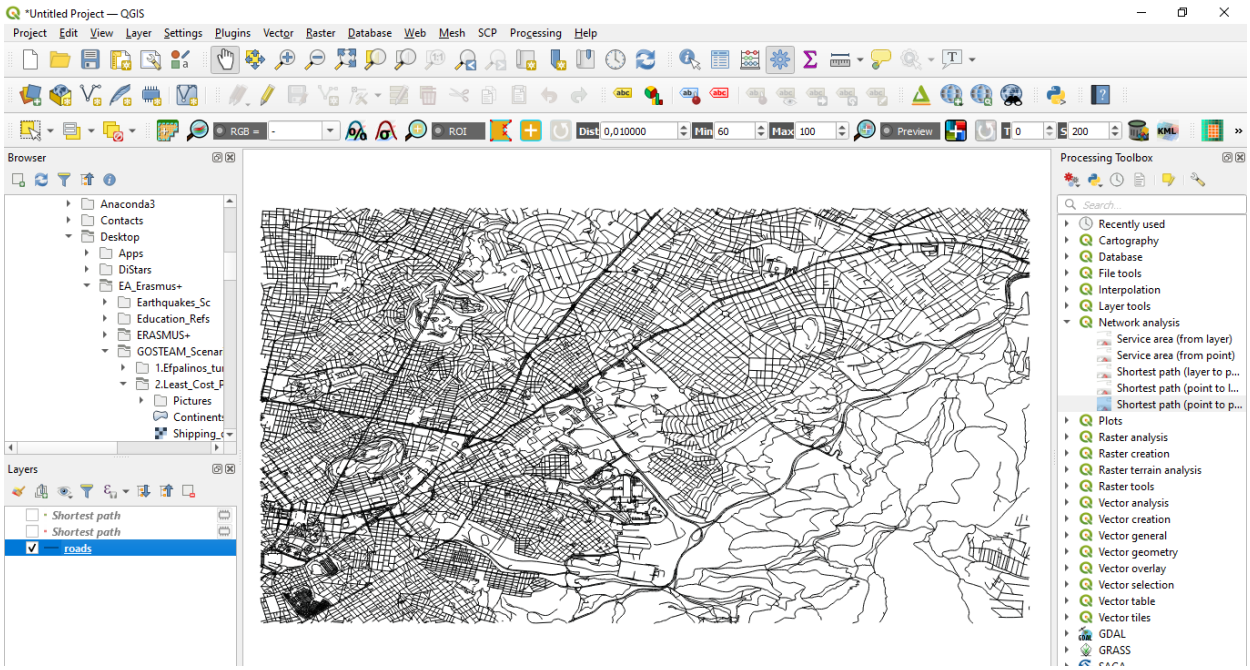


Figure 5: Data visualization using QGIS (using grey color)

But what do we exactly see????

We see that the road network of the area we've selected consists of numerous lines/segments (roads) and nodes (junction points between roads).

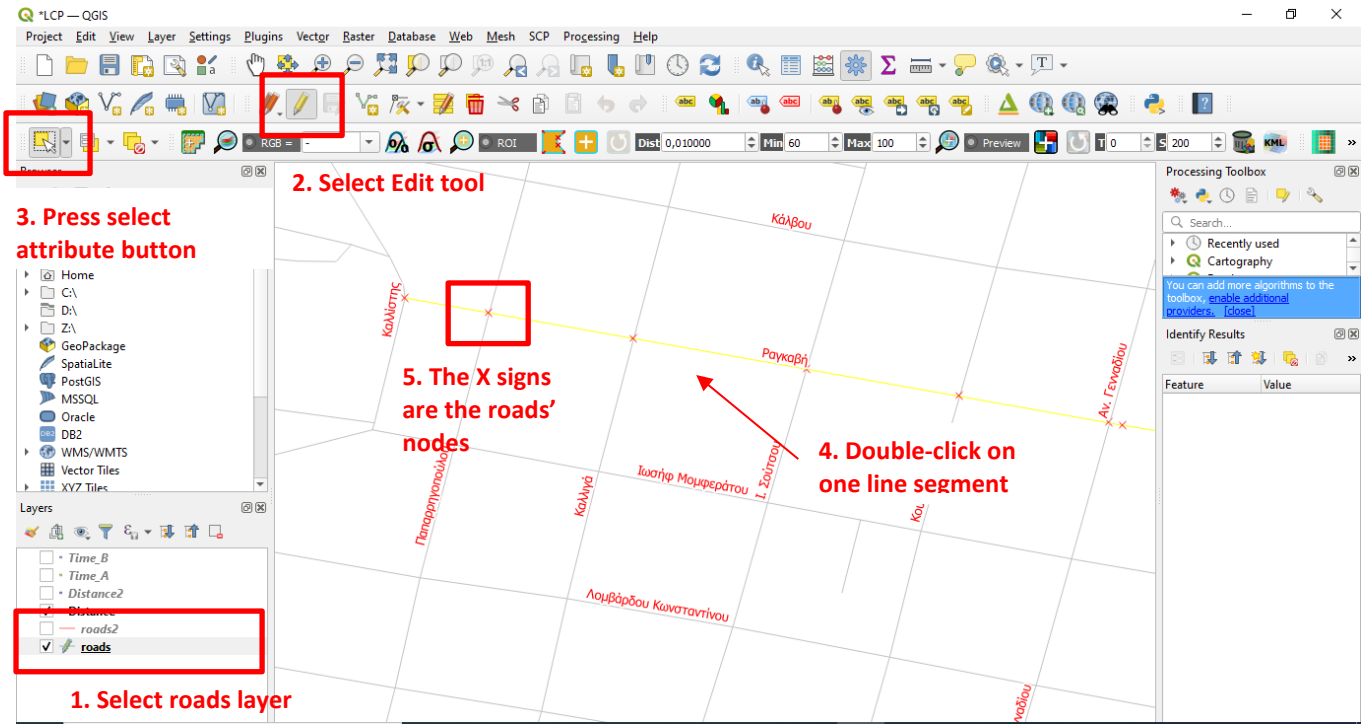
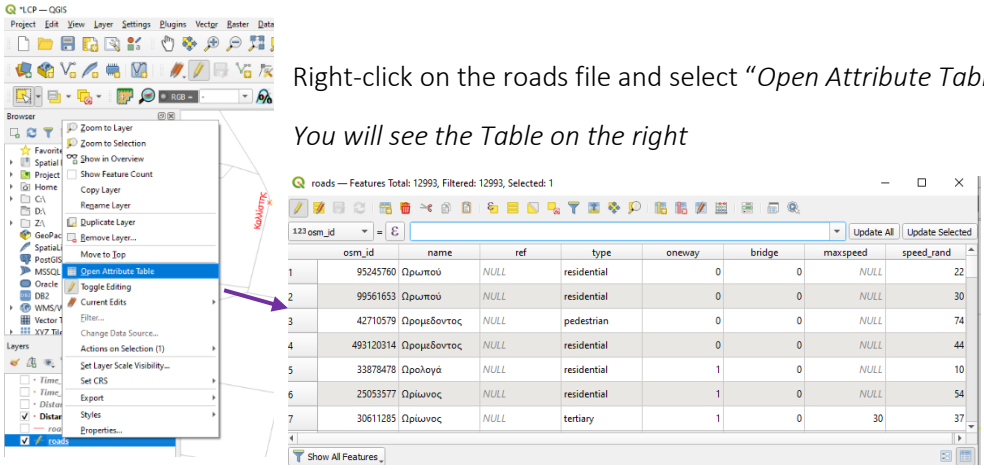


Figure 6: Roads' network using lines and nodes

It is important that the crossing segments are split by nodes! WHY? In order to know: i) the turning points and ii) in order to be able to add different qualitative or quantitative information for each segment (i.e. max. speed, length, name etc.).

Let's visualize roads' names on the map! But how the roads' names are stored to this type of data?

On the Attribute Table! See Figure 7.



Right-click on the roads file and select "*Open Attribute Table*"

You will see the Table on the right

Figure 7: Descriptive characteristics for each segment/line

Now, we can visualize the roads' names (See Figure 8)

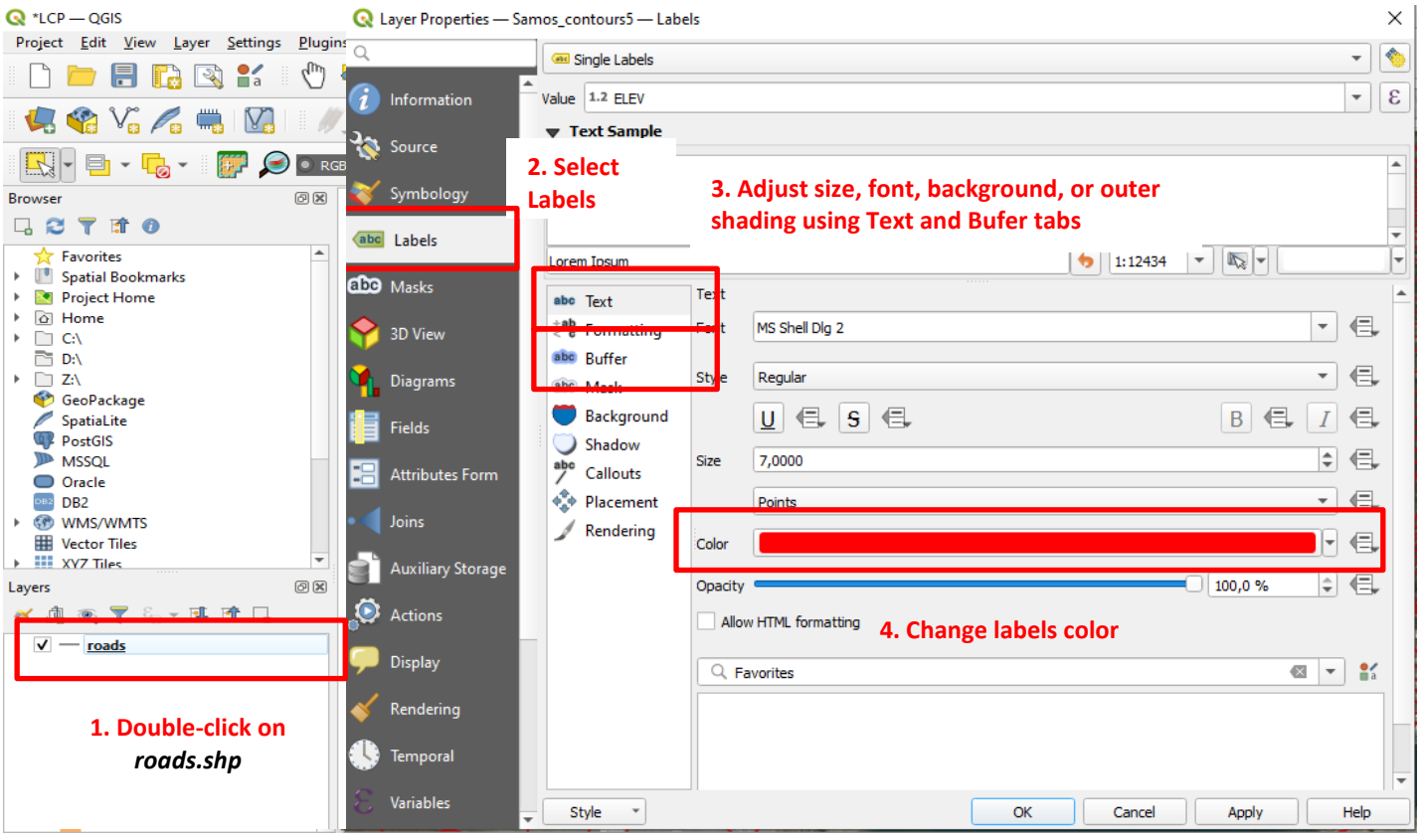


Figure 8: All appropriate steps you can follow in order to add labels on the map

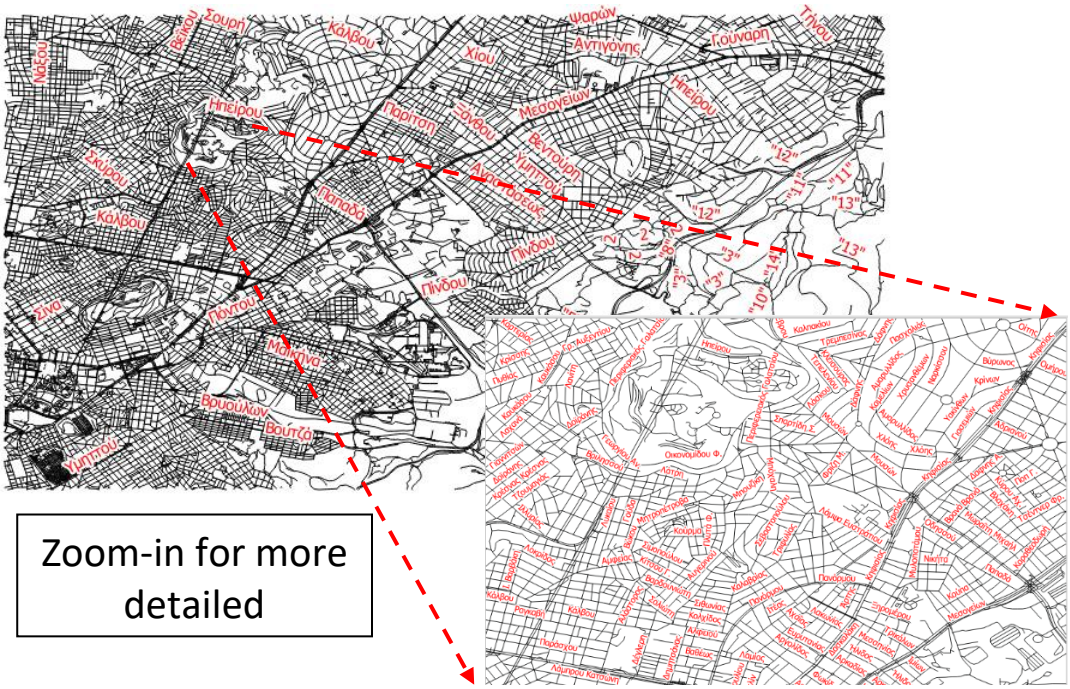


Figure 9: The results after adding the labels

Before we start analyzing paths, we must consider that we are using geospatial data, hence, all of these segments and nodes have a coordinate system!

Which coordinate system? Double-click on the roads layer and select Source!

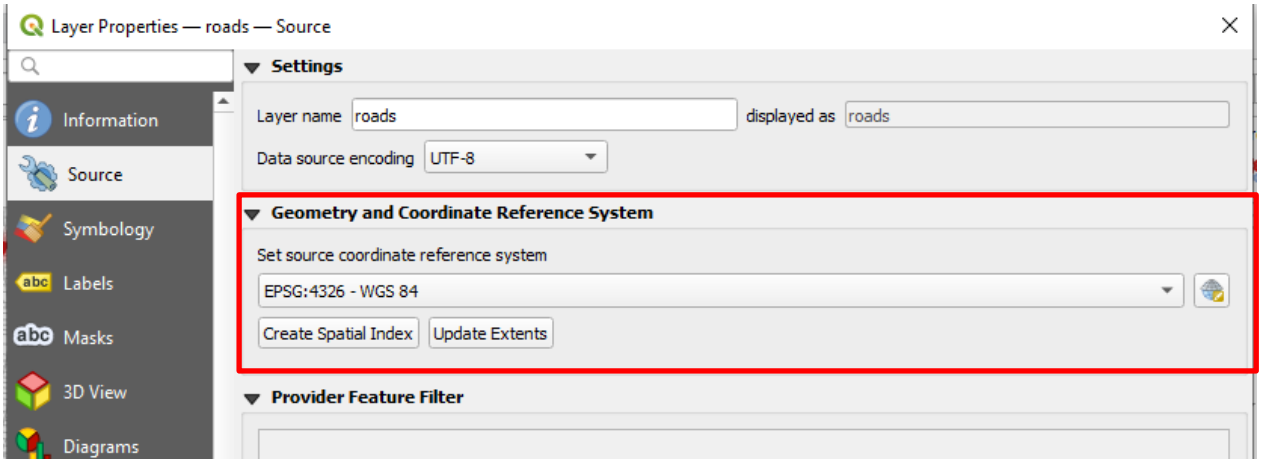


Figure 10: Coordinate System descriptive information

Search for EPSG:4326 on Google and try to explain what it is?

Read the first two results on Google. What units are used for the coordinates? How are the coordinates specified? Why is it called as a Geographic Coordinate System?

To change the coordinate system to a projected one, select on the main toolbar Vector > Data Management Tools > Reproject Layer and a new window opens (see Figure 11 on the right)

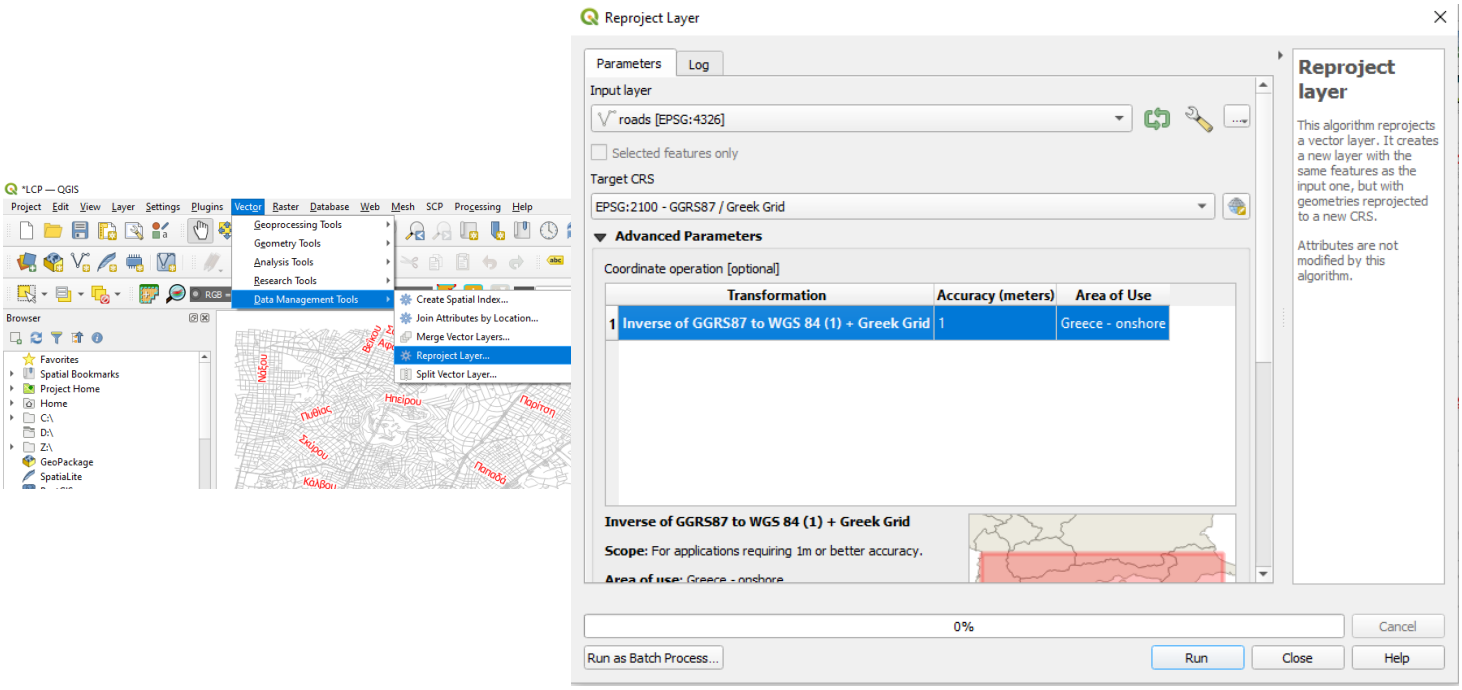


Figure 11: Transform to a Projected Coordinate System

On the new window (Reproject Layer) you select as input the ***roads*** file and as a target CRS (Coordinate Reference System) you can ask from student to select the projected coordinate system that is used in their country (e.g. for Greece is the Greek Grid, EPSG: 2100).

You save your file to your folder as ***roads_reprojected***!

Important note!!! Before you begin, go to the main toolbar and select Project > Properties > CRS > Search for the CRS you have already set in the previous Step and select it as your entire Project's layers CRS!
Why is that important? Because we want to estimate distances and times in meters and minutes, not in degrees!

NOW WE ARE READY TO SEARCH FOR OPTIMAL PATHS!!

To search for optimal paths we need specific algorithms. Here in QGIS to find these algorithms, we have to open the Processing Toolbox (see Figure 12, step 1) and go to Network analysis!

Why will we use Network analysis (Figure 12, step 2)???? Because we have already downloaded a road network with line segments and nodes!

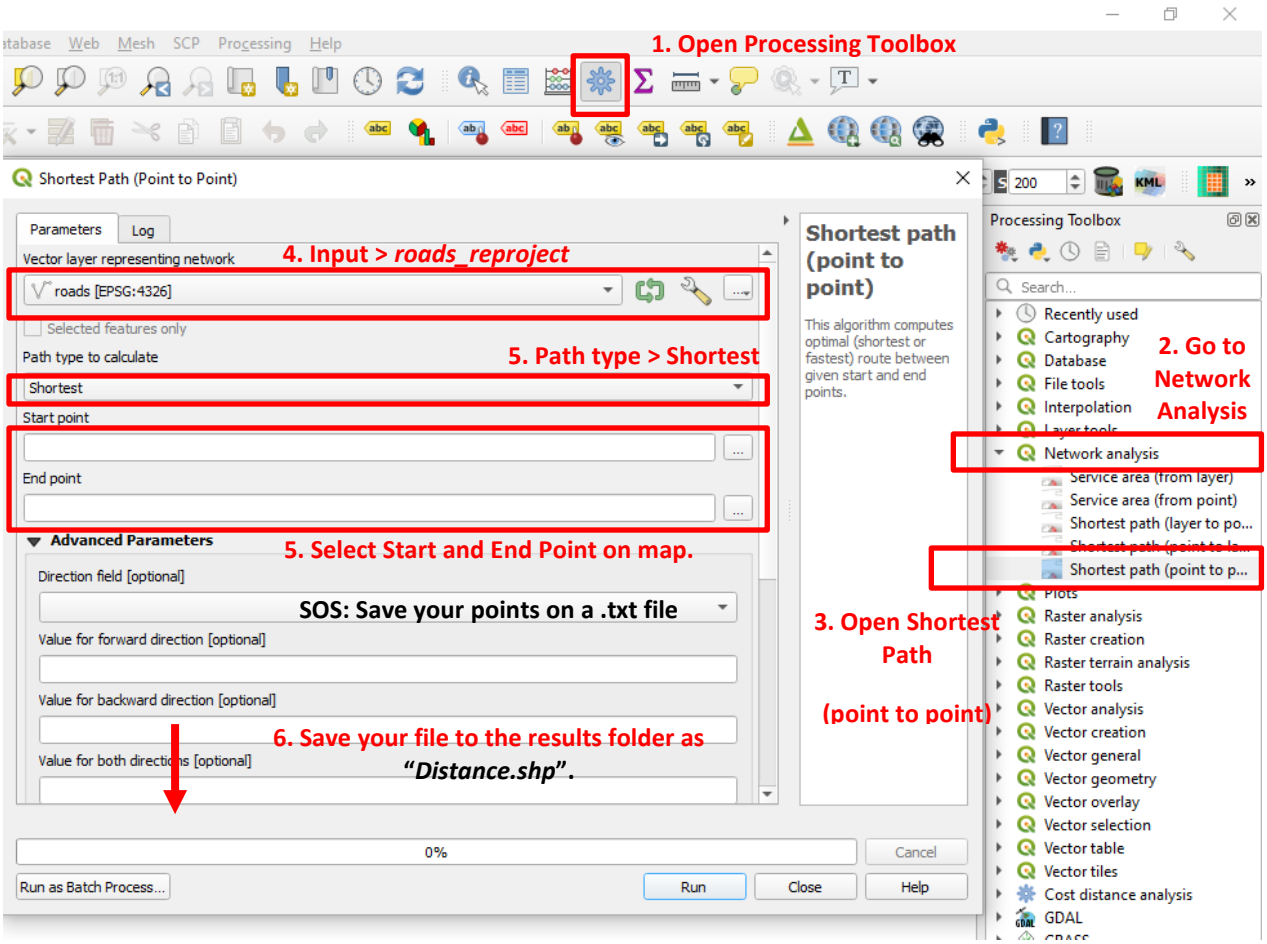


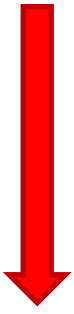
Figure 12: Steps for identifying optimal shortest paths

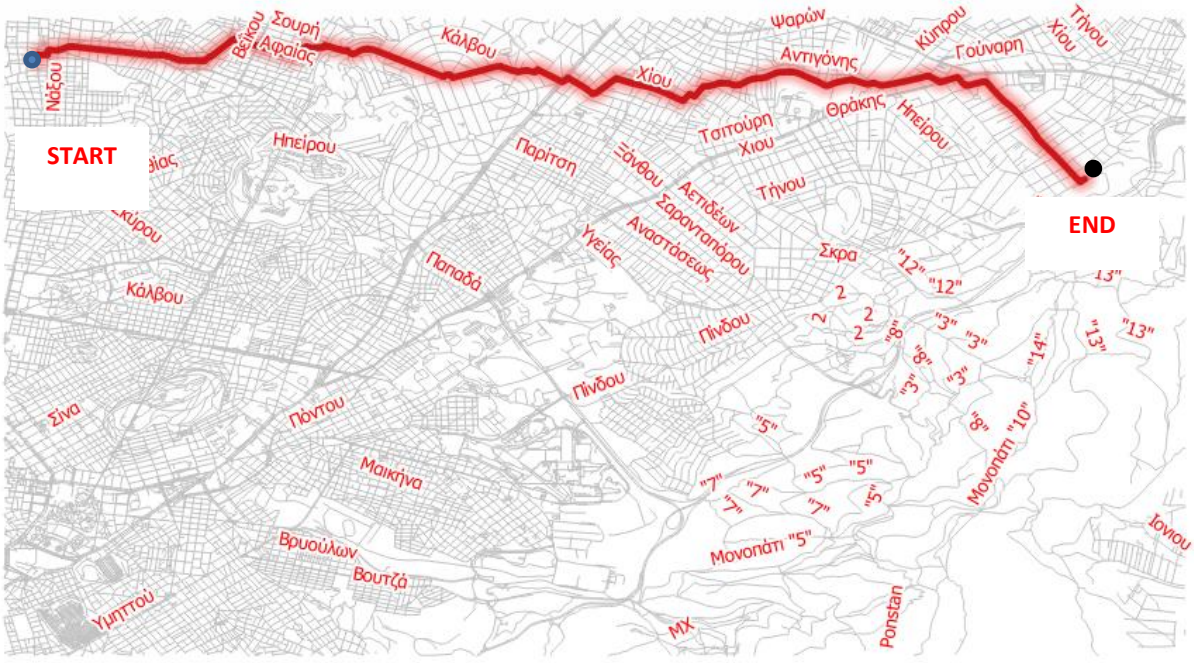
For extracting the shortest paths you must select the input vector layer (roads_repr), to set if you prefer the shortest or fastest path (here Shortest, see Figure 12, step 5) and most importantly to select a Starting and an Ending point (see step 5 in Figure 12).

You can set the Start and End points by clicking the three dots on the right of the white box, and just hit a point on the map!

Save you output file as Distance_path and you are done! DO NOT FORGET TO SAVE YOUR START AND END POINT ON A .TXT FILE!!

The Shortest path result (red path)





But what if we want to compare the shortest path with the fastest? What information we must consider? The answer is….speed and as a result, time!

How can we add this information? To the Attribute Table! This means that for each line segment (road) we must add the maximum speed limit!

To succeed that, follow the Steps of **Figure 13**!!!!

What do we want to do? To add a new field on the Attribute Table and set the maximum allowable speed per segment.

Therefore, right-click on the ***roads_reproject*** layer > Open Attribute Table > Toggle editing in order to be able to add a new column to the ***roads_reproject*** layer (see Steps 1-2, Figure 13).

Click on Field Calculator (Step 3, Figure 13) > Select create new field and name it *"speed_rand"* (Step 4, Figure 13)!

Why *speed_rand?* Because we are going to set a random speed for each segment rather than setting manually the appropriate speed for each segment.

Finally, type "rand(20,60)" on the Expression box which means that for each segment, a random value between 20 and 60 km. will be added.

rand(20,60) is a common programming expression. You can ask from students to search for similar random routines, for example, to select only floating random values at a specific range etc.

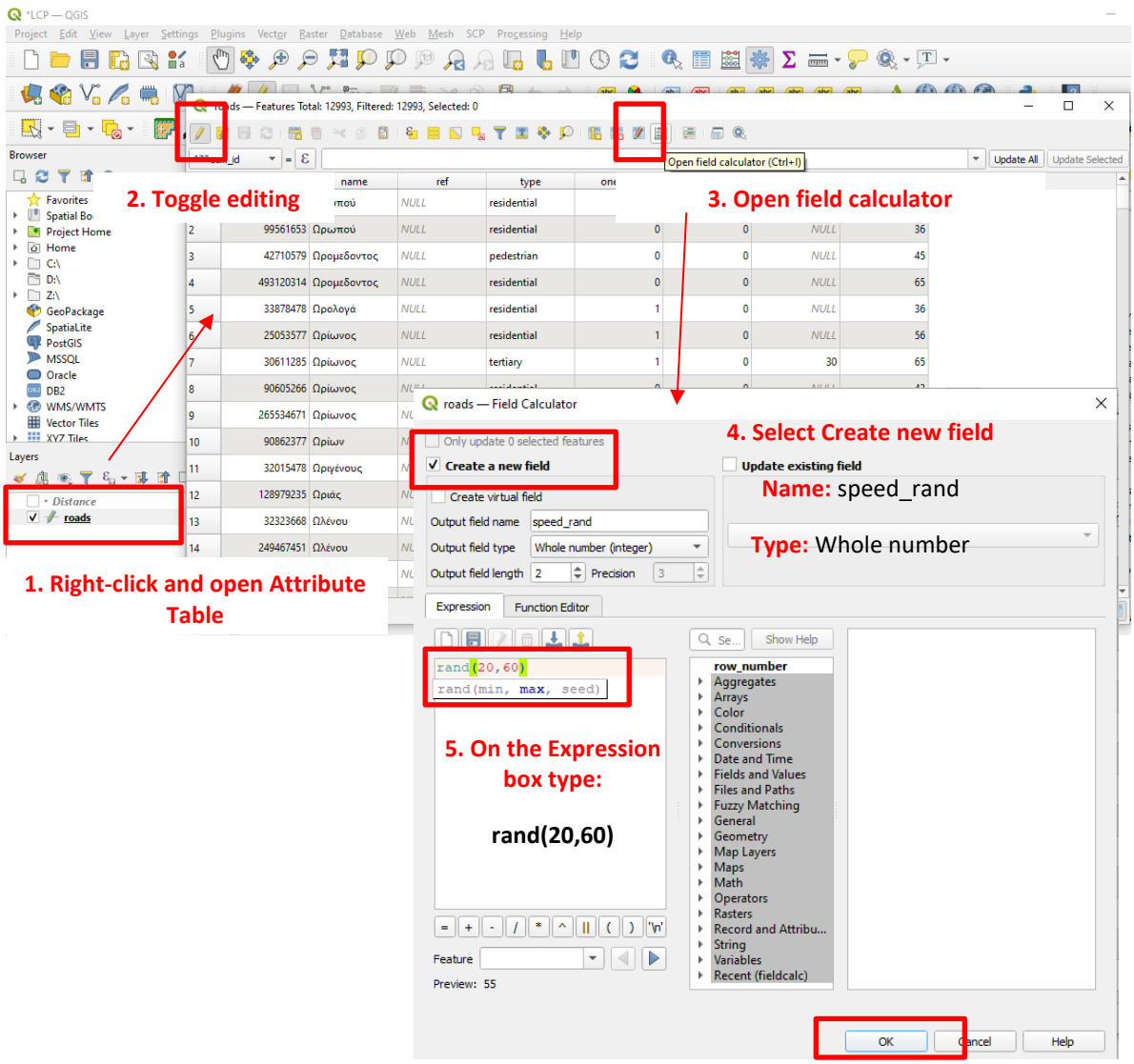


Figure 13: Steps for adding the information needed for the speed limits

After creating and updating the maximum speed column, you can run once again the Shortest path algorithm!

This time, you select as Path type > Fastest (Step 2, Figure 14) and the "speed field" of the Advanced Parameters (Step 4, Figure 14).

Important note! Set the exact same Start and End points that you have already saved to the .txt file.

Finally, save your layer as Time_A (Step 5, Figure 14)!

Compare the results!!! (Figure 15)

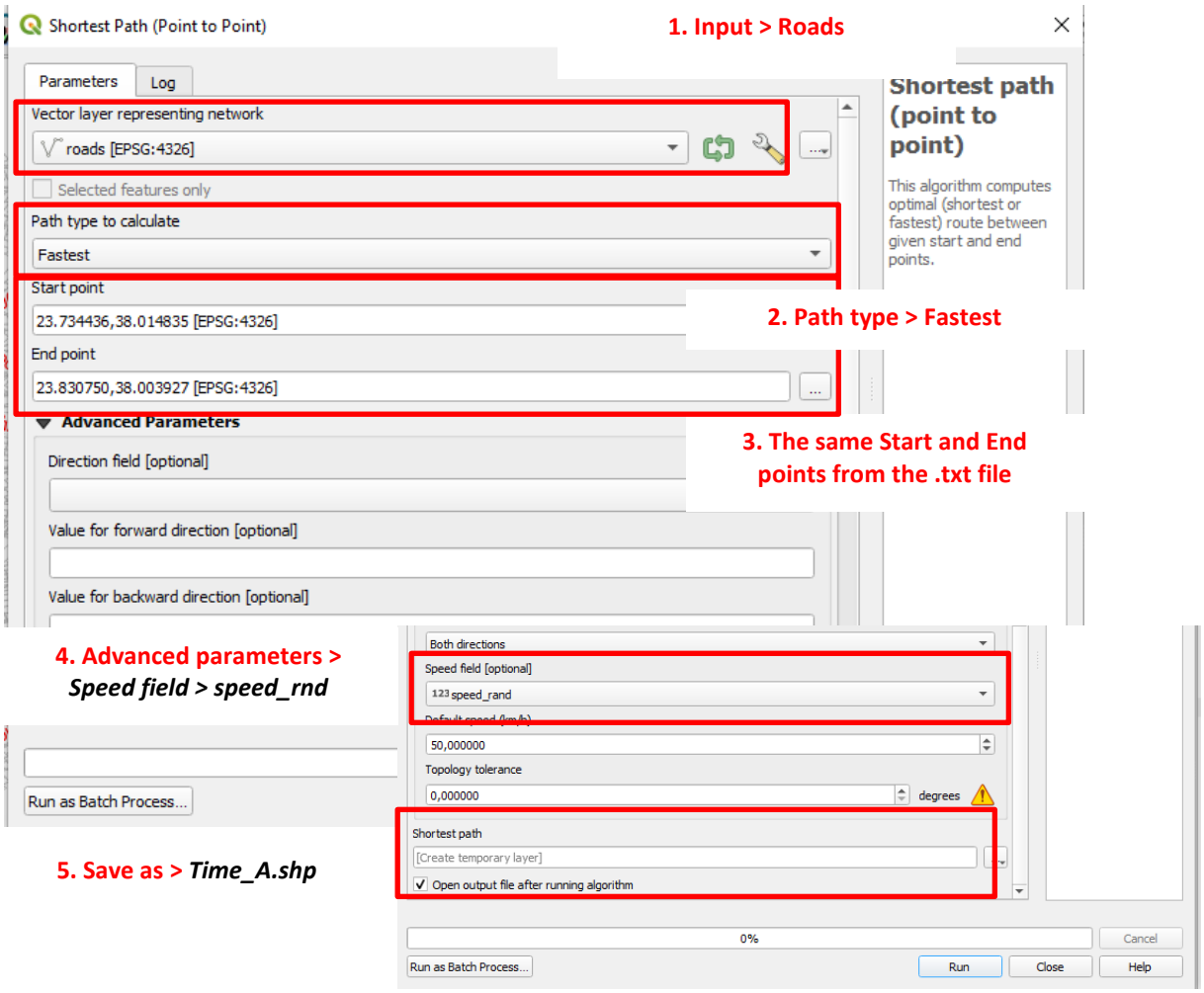


Figure 14: Steps for identifying optimal fastest paths

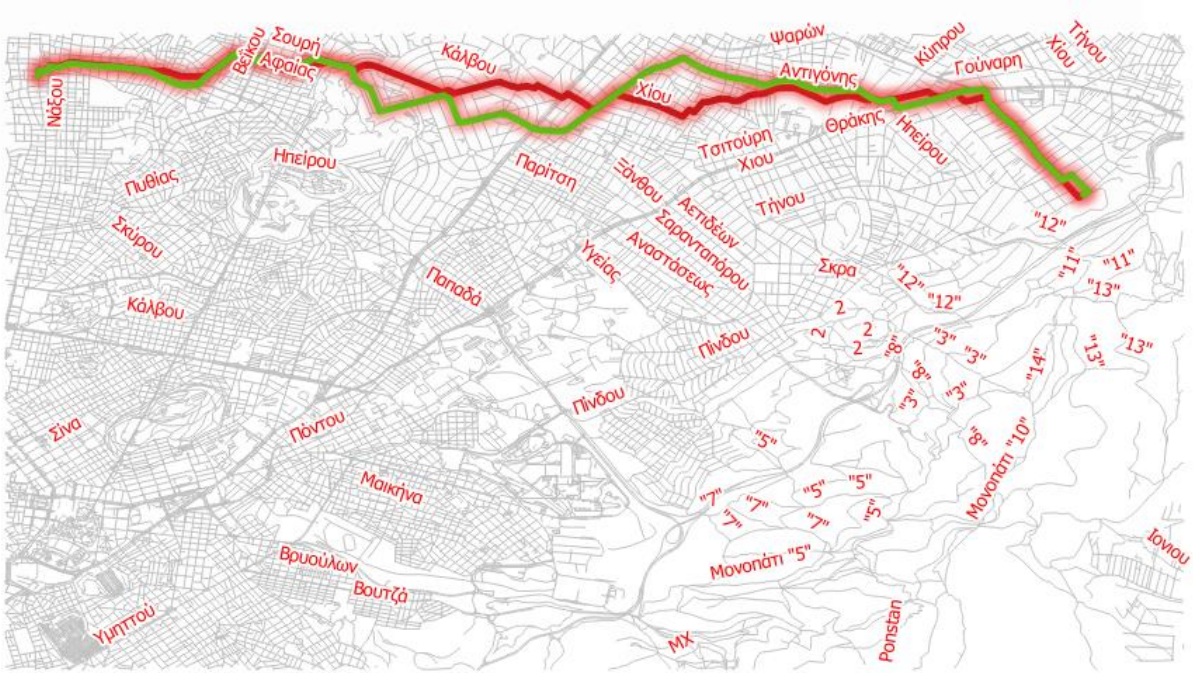


Figure 15: Comparison between the shortest and the fastest path

After comparing your results, you can test to update the speed limits of the *"speed_rand"* column! You can follow the steps of Figure 16 by Updating the values of an existing field on the Attribute Table!

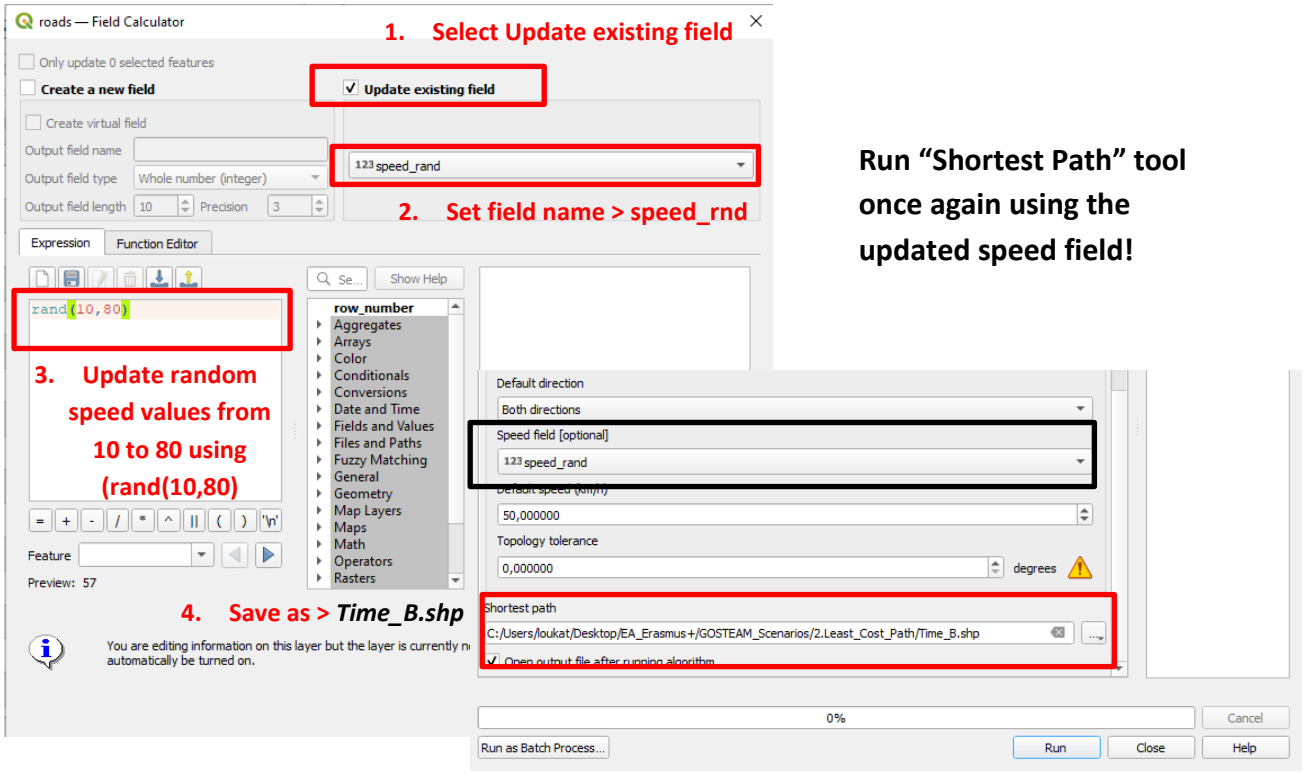


Figure 16: Steps for updating the information needed for the speed limits

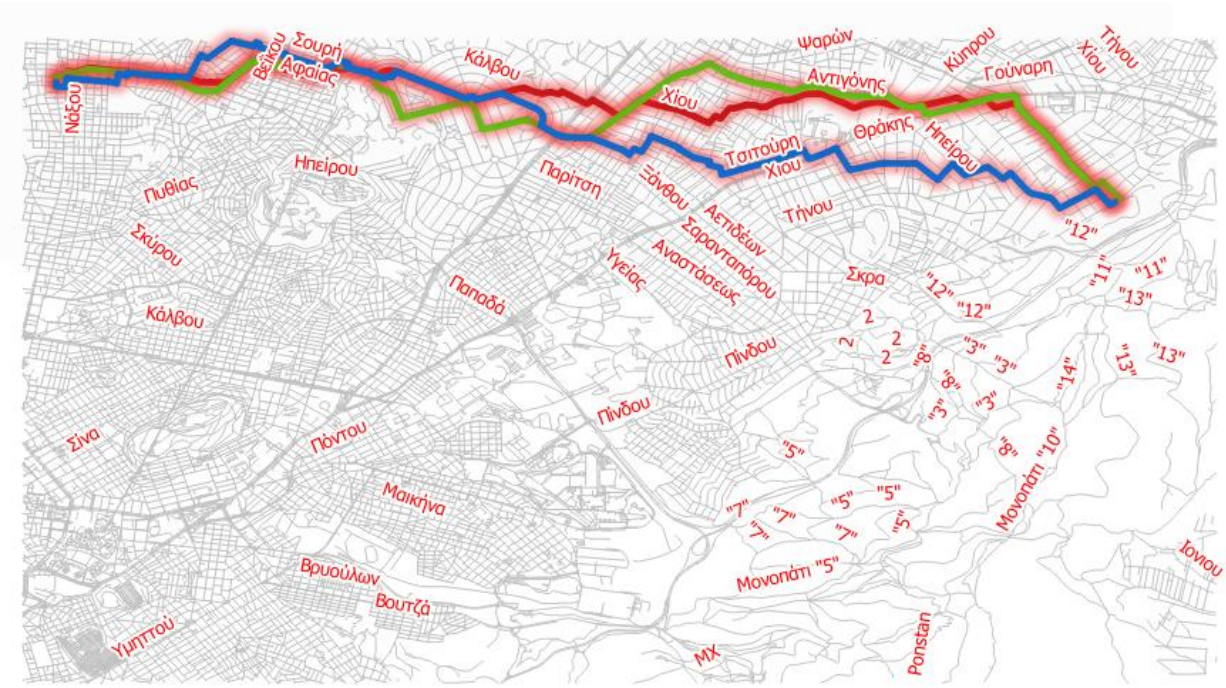


Figure 17: Comparison between the shortest and the fastest paths

By right-clicking on the resulted layers **“Distance”, “Time_A”** and **“Time_B”** you can open their Attribute Table and identify the distance covered (in meters) or the time spent (hours).

Advanced experimentation

As an advanced scenario of the Shortest path approach you can select to remove (clip) segments and nodes from your network! In real life, this can be linked to potential road network maintenance, traffic etc.

To succeed that, see Figure 17 and how you can select specific line segment using Select Features tool. After selecting the features you want to remove (yellow color in Figure 17) you right-click on the roads_reproject layer > Export and follow the instructions of Figure 17!

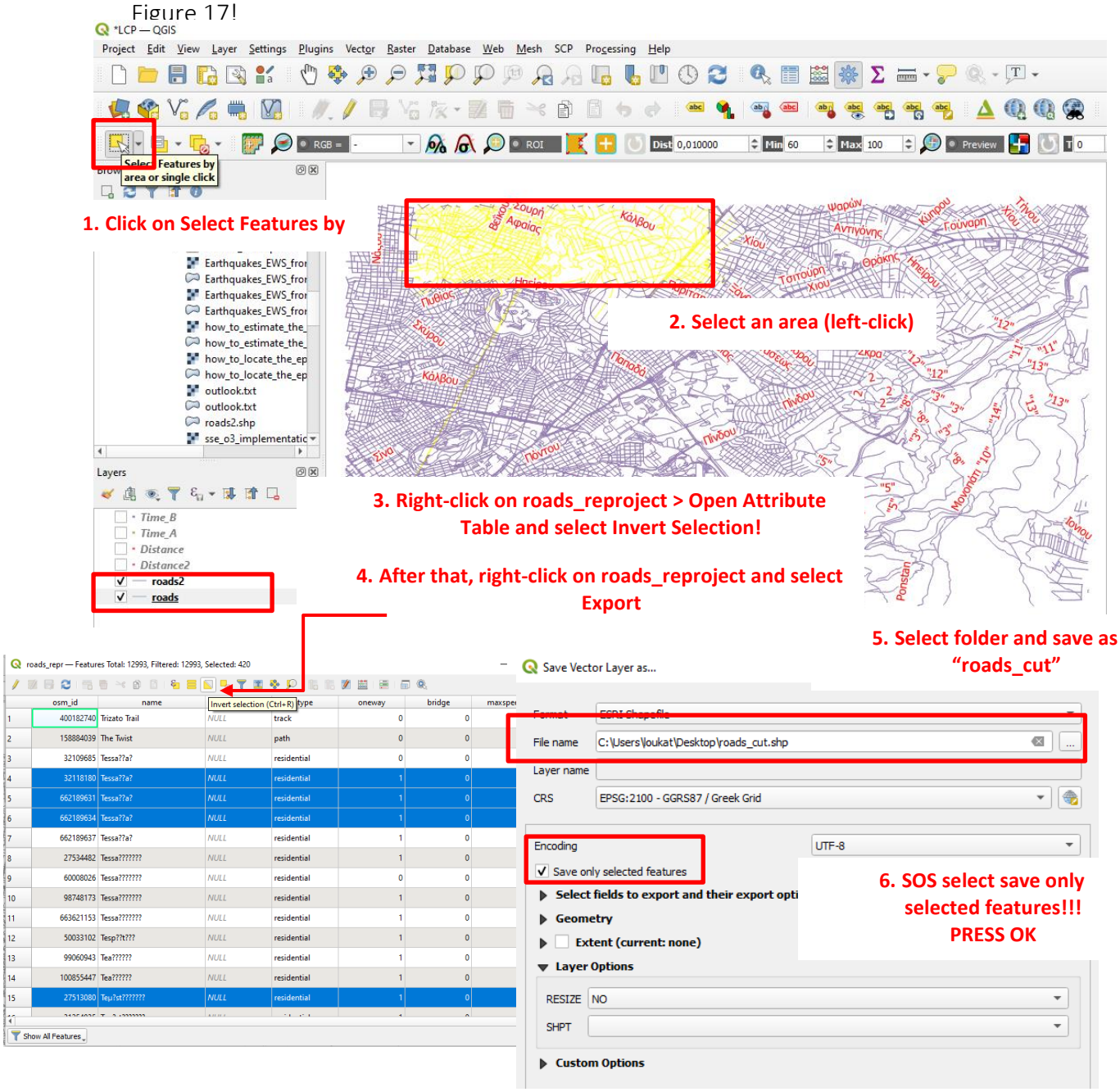


Figure 18: Select and cut features from the network

By following the procedure of **Figure 18** you will see your results (and the clipped area) as illustrated in **Figure 19**!

Repeat the same procedure to estimate the Shortest and/or the Fastest paths! Compare the results and the increase of the total distance covered or the time spent to drive from the Start- to the End- point!

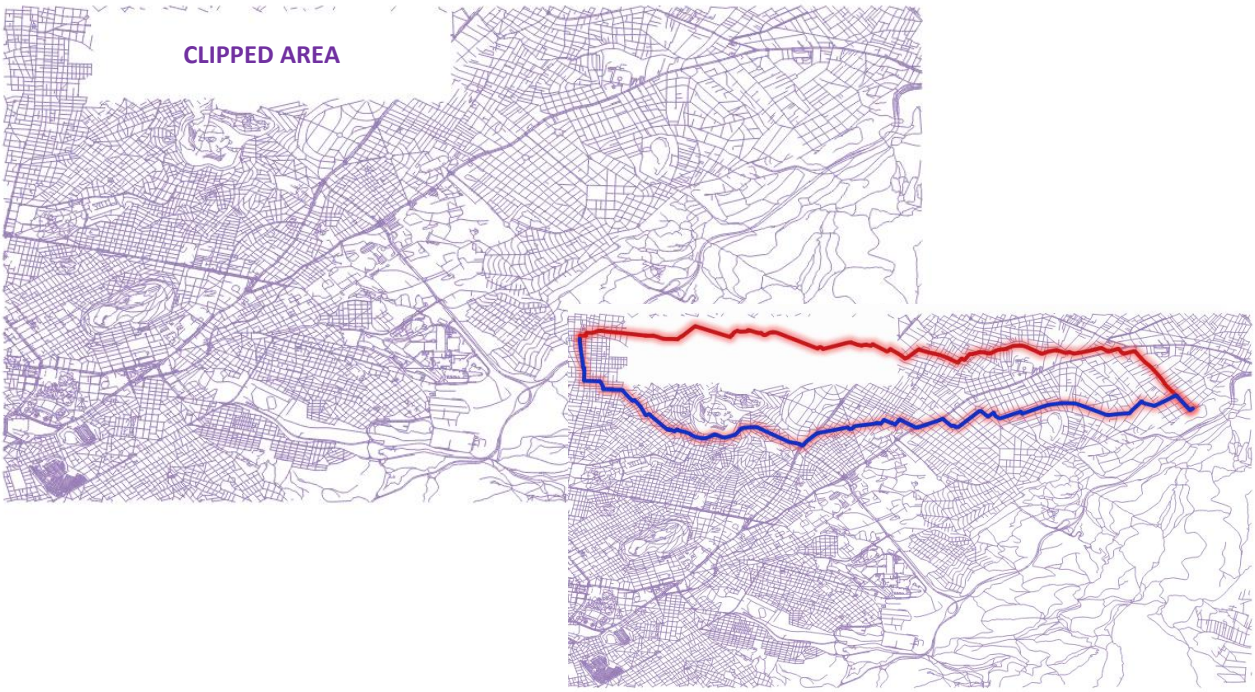


Figure 19: Final roads network after clipping a specific area (top left) and the resulted shortest path after running the Shortest path algorithm once again

Discussion

Explanation based on evidence

Guide the teachers to encourage their students to provide correct explanations for the topic(s) investigated.

Describe ways and they can use to this end and give them directions how to discover them.

You can ask from students if they can imagine the functions that are used in order to solve such a problem?

You can think of the examples we used in Figure 1!

Also, an analytical explanation is described here:

https://learningactors.com/minimum-cost-path-analysis-python/

Can they realize now the options that are provided to a user on the GPS????

For instance, why on the GPS we can select to drive using highways or excluding tolls and why the arrival time changes.

Consider other explanations

Facilitate the student groups to evaluate their own explanations in the light of alternative explanations, particularly those reflecting scientific understanding. Illustrate examples they can use and give them instructions how to locate them.

Which other criteria you think that might be used in Least Cost Path problems? For example, on the Google Maps navigation tool?

See the following video about the traffic monitoring:

https://www.priteshpawar.com/how-google-maps-works/technology-explained/priteshpawar/

What if we have to estimate the Maximum path or the Longest? (Advanced)

Just simply return the positive distances (i.e. weights) to negative right? Well, it is not that simple! See some examples:

https://stackoverflow.com/questions/23256534/maximum-profit-using-dijkstra-algorithm

https://en.wikipedia.org/wiki/Cycle_(graph_theory)

https://en.wikipedia.org/wiki/Directed_acyclic_graph

Reflection

Communicate explanation

Facilitate each student group to reflect on the previous experiences and to produce a report with its findings, presenting and justifying the proposed explanations to the other groups and the teacher.

Provide content which the teacher can use to help the students to get familiarized and to become efficient in scientific writing.

Ask from students to write a short report by incorporating the scope of this Activity (which is the objective?), the data and the tools they used, what methodology (algorithms) and to present their results in the form of maps (images) by reporting the Minimum distances and the total time needed.

They can also report all potential adjustments they made (clipping areas, speed estimation using random values) or any future research they want to consider (add some criteria or a simplified version of a Monte Carlo simulation considering the Fastest paths and the random speeds).

The report outline could be in the form of:

i) Introduction,
ii) Relevant information about Least Cost Path algorithms and the GPS,
iii) Study area and Data used,
iv) Methodology,
v) Results and Discussion and finally,
vi) Conclusions and Future work

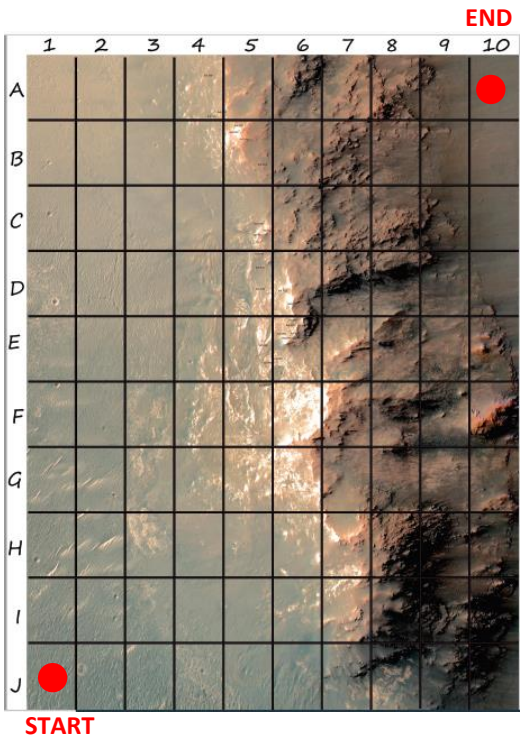
Follow-up activities and materials

Describe and direct the user to any follow-up activities or materials that can be used to wrap-up the hands-on activity.

These could include appropriate learning assessment and/or reminder materials (e.g., quizzes, games, other user-friendly tests), hints for further activities etc.

Return on the initial "Questions eliciting activities map" and ask from students to delineate again the shortest path from the START to the END point!

Tell them that they have to consider the criteria of the distance and the terrain slopes!



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References (if any):

Examples of LCP: https://gisgeography.com/least-cost-path-analysis/

Least Cost Path concept explanation: http://www.geography.hunter.cuny.edu/~jochen/gtech361/lectures/lecture11/concepts/Least-cost%20path%20analysis.htm

Route optimization planning and GPS: https://blog.routific.com/route-optimization-google-maps-313a45e13d27

Minimum Cost Path Analysis using Python: https://learningactors.com/minimum-cost-path-analysis-python/

Cycles and Graph theory: https://en.wikipedia.org/wiki/Cycle_(graph_theory)

Directed acyclic graph explanation: https://en.wikipedia.org/wiki/Directed_acyclic_graph

Assessment (if any):