GOSTEAM Hands-on Activity Template *(Inquiry-based)*

Title:

Trilateration and the GPS

Short Description (Max 500 words):

Students will be exposed to the principles of trilateration (Geometry and Physics) and its application to the most modern way of finding your location (2D and 3D intersection points, coordinates and estimates uncertainty). Students will simulate the principles of the GPS using an interactive map in order to locate their position using a number of fictional satellites. Nevertheless, how accurate is a GPS measurement?

Keywords (Up to 5):

GPS, Satellites, Trilateration

Information about the Implementation

Age and language of the students: 9-12 12-15 15-18 18+

Language: Greek Age: ☐ 9-12 ☑ 12-15 ☐ 15-18 ☐ 18+

Number of Lessons – Duration (per lesson):

Number of Lessons: 1 Duration per Lesson: 45’

Subjects:

For which subject(s) the activity is usable, is it an interdisciplinary activity?

Science ☑

Physics ☐ Chemistry ☐ Biology ☐ Geosciences ☑ Environmental ☐ Other ☐

Technology ☑

Engineering ☑

Arts ☐

Mathematics ☑

Information about the Scenario

Curriculum and country:

Link of the current activity to the curriculum:

Country: Greece

Class: Secondary

Grade: 7 - 10

Topic: 2D – 3D Surfaces, Spatial Analysis, Surface characteristics

Objectives (Max 100 words):

Description of the learning objectives

Some of the key objectives to be addressed through this activity include:

1. Introduction to the science behind the GPS.
2. Exposure to the principles of trilateration and its application to the most modern way of finding your location.
3. Simulate the principles of the GPS using an interactive map in order to locate their position using a number of fictional satellites.

Materials (Max 100 words):

Which resources and materials (software, hardware) are needed?

Material: Search Engines, Satellite Images

Software: QGIS (qgis2threejs)

Spatial concepts, skills, and abilities:

Which spatial concepts and skills are covered by the activity?

Spatial concepts:

Primitives: Identity/Name ☑ Location ☑ Space/Time ☐

Simple: Distance ☐ Direction ☐ Connectivity ☐ Movement ☐ Boundary ☐ Shape/Area ☐ Adjacency ☐

Difficult: Overlay ☐ Buffer ☐ Topology ☐ Coordinate ☐ Map ☐ Scale ☐ Shortest Path ☐ Navigation ☐ Surface ☐ Slope/Gradient ☐ Aspect ☐ Contour ☐

Complex: Interpolation ☐ Map Projection ☐ Spatial Dependency ☐

Other:

Spatial skills:

- [ ] Map literacy
- [ ] Navigation/orientation
- [x] Estimating distances and directions
- [ ] Recognizing and understanding patterns/Understand and identify models of spatial organization
- [ ] Select an ideal location based on the given spatial features
- [x] Visualization
- [ ] Understand and identify spatial correlations/ dependencies
- [ ] Categorize spatial entities/ geographic features and identify hierarchies
- [ ] Compare spatial entities and draw analogies among them
- [ ] Identify/determine connections/relations
- [ ] Understanding scale in space and time
- [ ] Delineation of spatial regions/ zones based on given features/ properties

Short Description

Navigation/orientation: Finding one's way in unfamiliar environments, interpreting and giving walking and driving directions.
Estimating distances and directions: Measure paths, weighted distances, angles.
Map literacy: Using, interpreting/understanding, learning from, and communicating acquired spatial knowledge from maps, comprehension of geographic features represented as points, lines, or polygons.
Recognizing and understanding patterns/Understand and identify models of spatial organization. Delineation of spatial regions/zones based on given features/properties: Regionalization processes, pattern recognition and clustering identification in the 2d and/or the 3d world.
Select an ideal location based on the given spatial features: Single or multi-criteria siting and optimal areas identification.
Visualization: Visualizing spatial entities from written/oral verbal descriptions, from their 2d or graphical representations or through mental transformations; such as axis rotation or perspective taking.
Understand and identify spatial correlations/ dependencies: The ability to realize, identify and explain patterns, clusters and relevant spatial dependencies.
Categorize spatial entities/geographic features and identify hierarchies: Identify the hierarchical form of data and gradients between spatial entities.
Compare spatial entities and draw analogies among them: Calculate and compare different geometric objects' shapes, area and boundaries.
Identify/determine connections/relations: The ability to identify links and common characteristics among spatial entities and between humans and spatial entities.
Understanding scale in space and time: The understanding of changes/transitions through space and time for different spatio-temporal scales.

Geospatial concepts and spatial abilities documentation (see Section 3.2):
http://www.gosteam.eu/wp-content/uploads/2021/05/GOSTEAM_IO1_A1_final.pdf

Description of the activity in detail

Question Eliciting Activities

Provoke curiosity

Describe ways and materials that teachers will present to their students to attract their attention to the topic investigated.

- *Usually, the most effective way to provoke students' curiosity is by presenting an exciting video or a series of photos*

Watch the following video (up to 2:07) to get introduced to the functionality of the GPS:

https://www.youtube.com/watch?v=RSA3feQ9gKk

You can explain in parallel the description below, focusing on the history of GPS and the fundamental questions arising while describing how GPS works.

The first question that comes into mind is: "How does the GPS work? And how does it achieve such a tremendous impact on our lives?" (Figure 1)

Nowadays, everyone who wishes to find their location on a map or ask for directions concerning the navigation towards a specific destination, use the: Global Positioning System (or in short: GPS).

Figure 1: GPS devise used for navigation purposes

Originally constructed in the 70's by the USA Department of Defence for military purposes, the GPS can be found everywhere: from cell phones, to bulldozers, wristwatches and other gadgets. Every technological unit which needs precise timing and location information uses GPS. Scientific instruments, military equipment, communication networks, transportation, financial markets and power grids are only a few examples of applications depending heavily on GPS for precise time synchronization.

Using a constellation of GPS satellites (31 of them up to August 17, 2015), every location on earth is simultaneously monitored by 5 satellites so that one can obtain a stigma of their location with an accuracy that can reach 3 to 10 m. Considering the fact that the satellites are orbiting at a 20,000 km distance from the earth's surface you can imagine the precision of these instruments!

Propose preliminary explanations or hypotheses

Formulate the scientifically oriented questions that teachers will present to the students to trigger their engagement in thinking about the topic investigated based on their existing knowledge. Make these questions digitally available and easily usable, e.g., by integrating them in the materials described in the previous step.

It is best to ask these questions in the context of a relative discussion.

Trilateration is a general geometrical method which has numerous applications. Once the students have gotten to comprehend the concept of trilateration using this educational scenario, they can easily jump to activities such as locating the epicenter of an earthquake and others. For further information concerning trilateration and the GPS you can visit the following links:

https://www.youtube.com/watch?v=4O3ZVHVFhes (Trilateration explained)

https://www.youtube.com/watch?v=PLjld-edVj8 (3D trilateration)

Make sure that the students do not confuse trilateration with triangulation. Trilateration is defined as the method employed in order to find your unknown location when you know its distance from at least three reference points. Triangulation is the method employed in order to calculate the unknown distance between a reference baseline and the desired location, with the knowledge of the baseline length and the angles between the location and the two ends of the baseline.

https://www.youtube.com/watch?v=Nv_oiLPJOV0 (What is triangulation)

The GPS works using the method of trilateration. Let's see what this is all about: Trilateration is the method used to find the unknown location of an object when its distance from at least three objects is known. Trilateration (or more so quadrilateration as we are talking about 3 dimensional objects) is fully employed in the GPS and has numerous applications such as finding the epicenter of an earthquake.

How is it done? This is the centerpiece of our investigation!

Active Investigation

Plan and conduct simple investigation

Provide the teachers with a specific plan of the investigation that will take place. Offer instructions about the activities they students will need to perform and what kind of materials they may need. Describe ways that the teachers can use to facilitate the students to focus on evidence.

This is the phase in which students are being prepared for the subsequent phase of evidence gathering during observation.

In order to comprehend in depth, the principles governing the function of the GPS, we need to review some fundamental geometry and physics concepts:

Intersection of two circles

Two circles of different radii can intersect having thus two points in common:

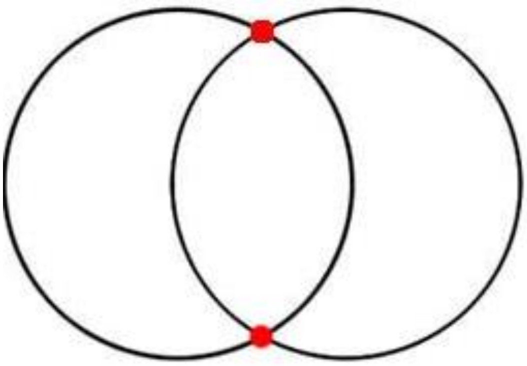

Figure 2: Intersection points between 2 circles

If we go to 3 dimensions, two spheres intersect in the following way:

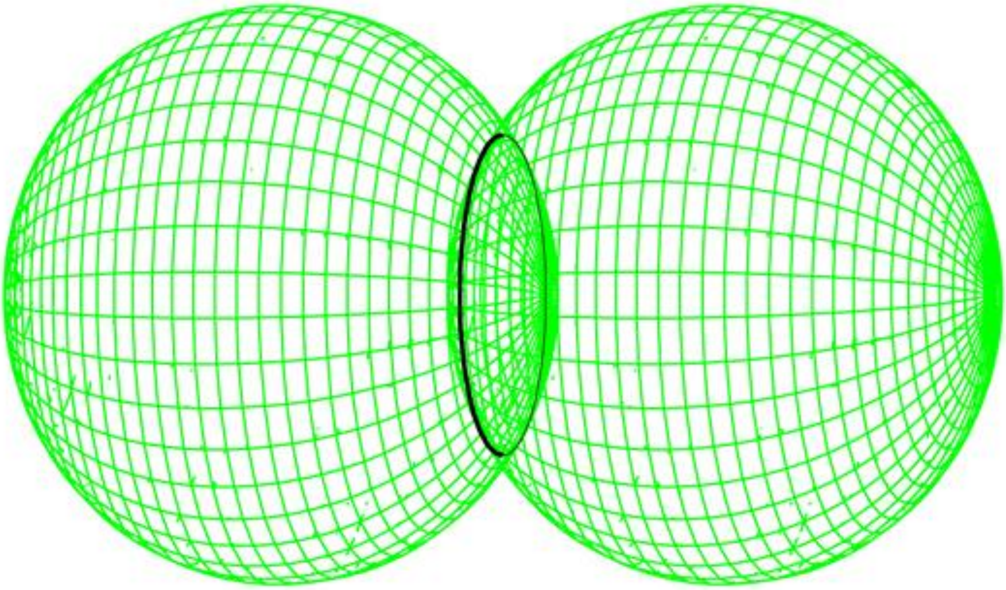

Figure 3: Intersection points between 2 spheres

Can you define the common territory between the two spheres? Is it a point or something else? What would happen if we had a third sphere intersecting with the two?

How many common points would there be?

Uniform Linear Motion

Let's start by remembering the all time classic relationship for uniform linear motion – i.e the motion of an object with constant velocity.
Time, distance and velocity are related by the following equation :

$$x = u \cdot t$$

In order to find the value of one of the three variables, the other two must be known from before. Can you think of some everyday examples in which we observe uniform motion? Discuss them with your classmates.

The speed of Light

Light travels with constant speed, which differs from the one medium of propagation to the other. In vacuum, the speed of light is:

$$c = 299792458\ m/s$$

In other mediums, the speed of light equals to the value stated above, divided by a number which is larger than 1 and differs from medium to medium (it is called the index of refraction and depends on the properties of each medium).
The speed of light in vacuum is the highest speed in the Universe, and is a constant. This constancy is very important for "Time of Flight" measurements. When we want to measure the distance between two points, a simple way to do so is to place a light source in the first point and a detector in the other point. If we measure the time needed for the light to go from A to B, then we can multiply by the speed of light in order to obtain the distance between the two points.

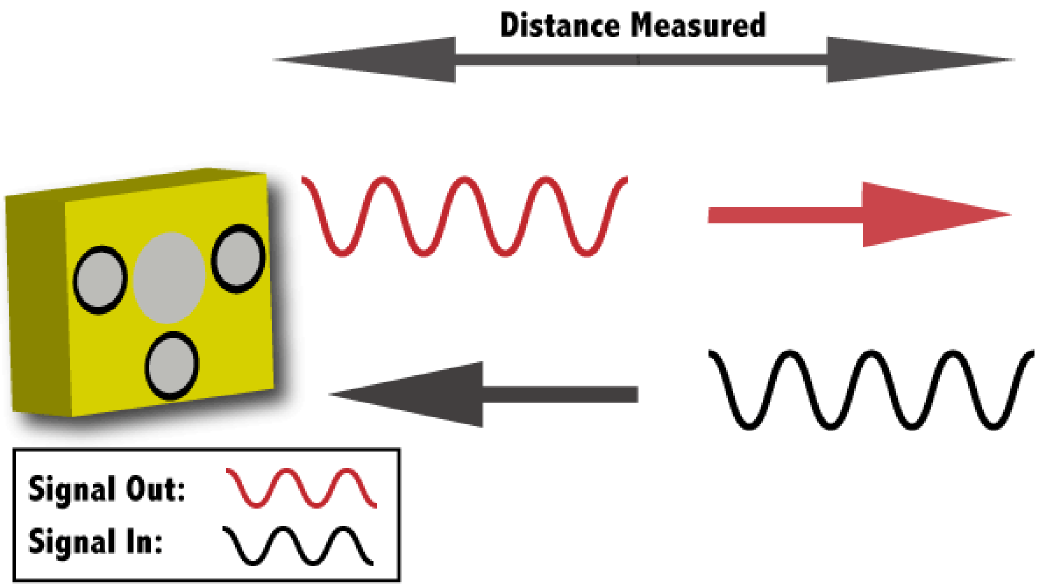

Figure 5: Using signals to obtain the distance between the two points

The above picture shows a more realistic implementation of the time-of-flight principle. A light source (yellow box) emits a light pulse which reflects on the black object. A clock starts counting time from the moment of the emission and on. The light reflects and goes back to the yellow box where it is detected. Its arrival time is recorded.
If we define the unknown distance between the two objects as X , the total time elapsed between emitting and receiving the signal at this example is:

$$t = \frac{2X}{c}$$

Thus, after measuring time and replacing the known value of the speed of light, we can obtain the value of the distance. (The factor two appears because light travels the distance twice, once to and once from the black object).

Can you think of any everyday application of the time-of-flight technique?

What do you think? Is it possible to perform the same technique without using light but another probe instead?

Discuss your views.

Creation

Gather evidence from observation

The selected resource (e.g., a simulation, an experiment, an animation, a graph, or other exhibit of similar nature) must provide students with an opportunity to collect evidence addressing the scientific questions presented in previous stages through direct or indirect observation. Provide guidance to the teacher organize and manage the activity most effectively and efficiently.

It is recommended to introduce group work at this stage. Divide students in groups, each of which will be facilitated by the teacher to formulate and to evaluate explanations to the scientific questions based on the collected evidence.

Divide in 3 groups. Each of the groups will be given the same coordinates of the GPS satellites' positions and different data that will be needed in order to calculate the distances between the satellites and your location.

As a first step of your investigation, browse the interactive map here:

https://mapmaker.nationalgeographic.org/

Get used to the controllers and the options of the map.

Click on the Latitude-Longitude options and choose to Show Longitude and Latitude position as demonstrated on **Figure 6**.

Using the "Draw a marker" option you will flag the GPS satellites on the map (**Figure 7**)

Each satellite should be flagged with a marker of different number.

Make sure that you pinpoint the satellites at the exact spots provided.

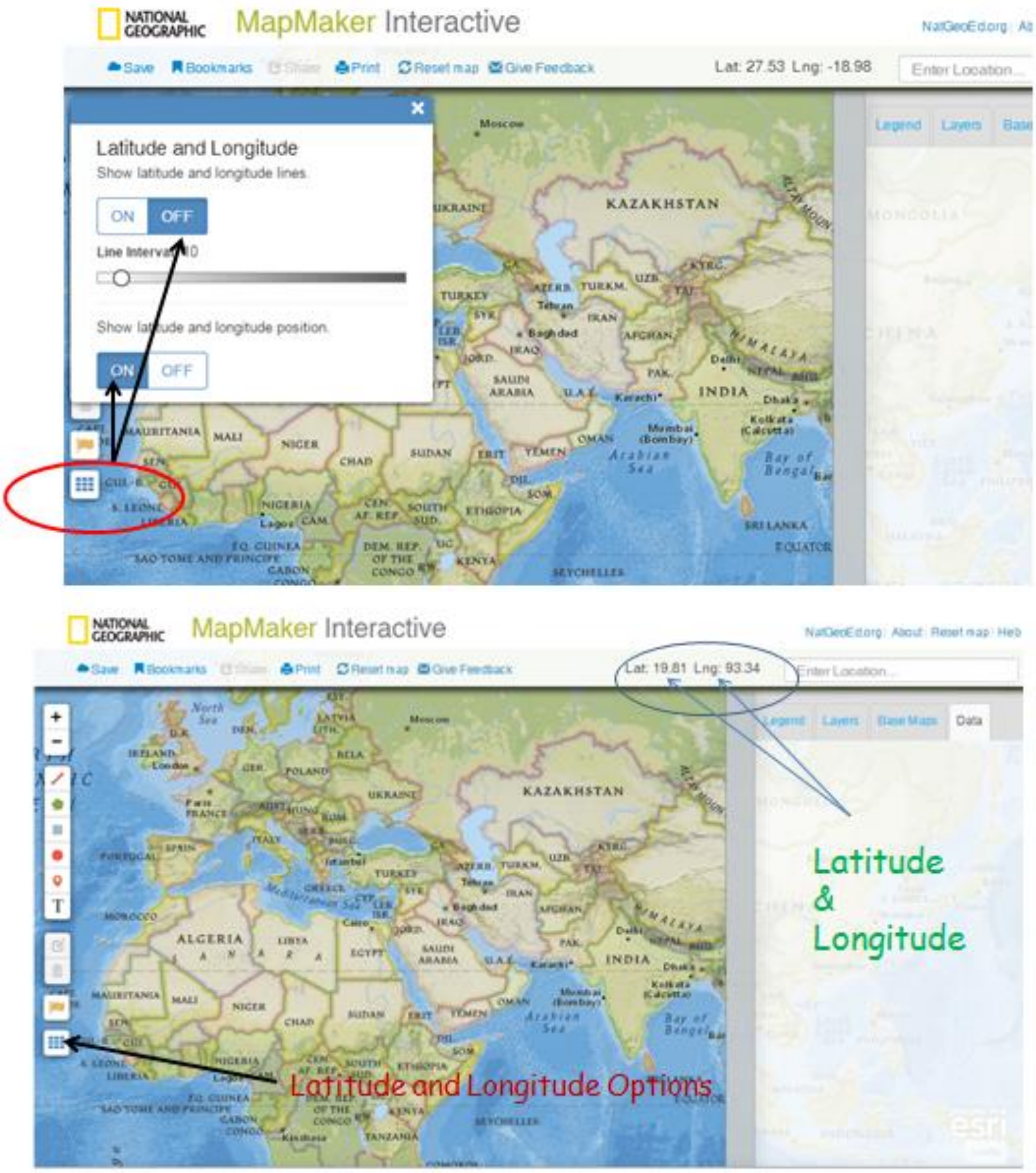

Figure 6: Enable and identify Lat – Lon information

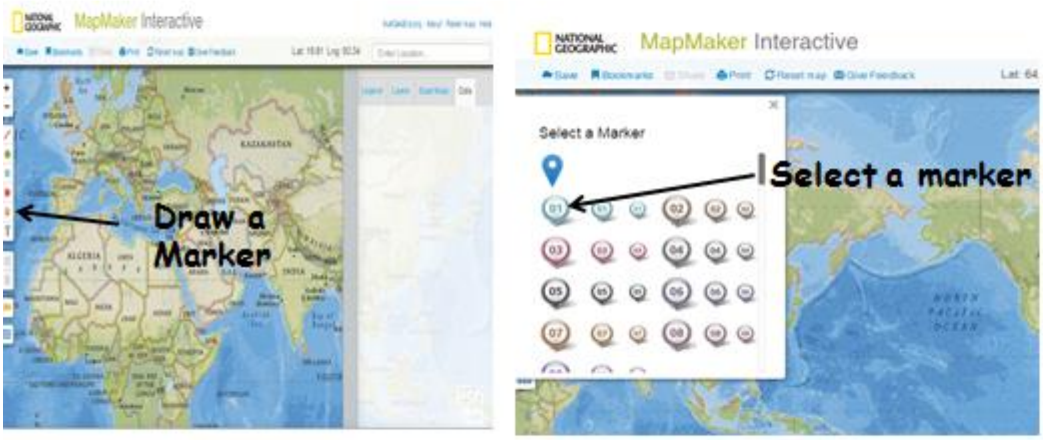

Figure 7: Drawing a marker and save in parallel the Lat-Lon

Latitude and longitude will be provided to improve accuracy. If you make a mistake, then choose the option "Reset Map" and start all over. Use the zoom in and zoom out option for extra help. Examplary locations of the satellites are given at the following table:

Satellite Nr	Latitude (◦)	Longitude (◦)
1	41,82	4,31
2	54,53	1,65
3	49,72	-18,5

After you have flagged the GPS satellites, your map will look like this:

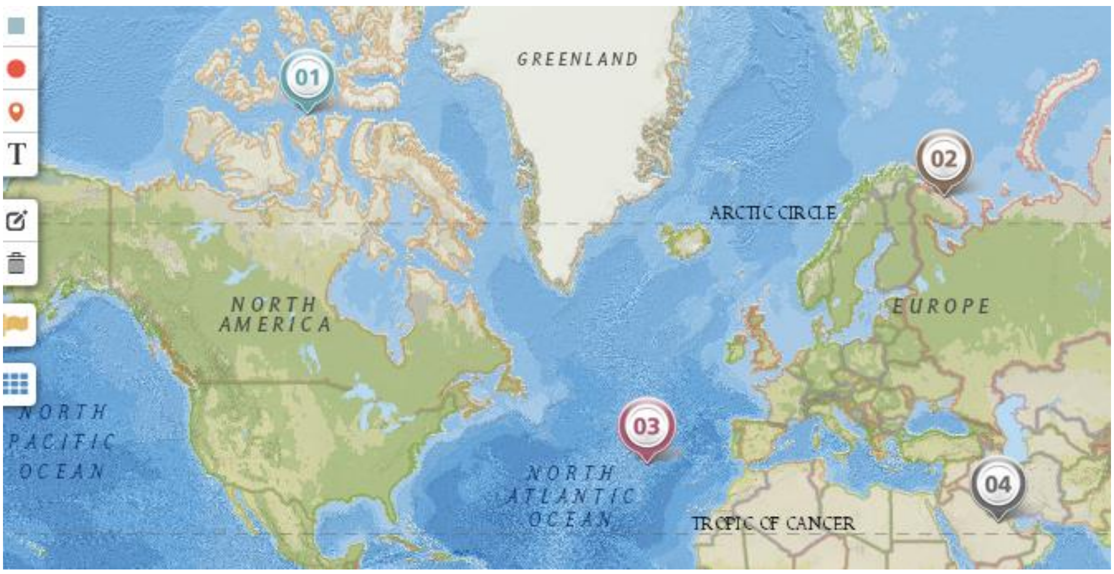

Figure 8: Final digitized satellites locations

Perform investigation

Now each team shall use the following data concerning the time elapsed from each satellite.

Team 1:

Satellite	Time Interval (sec)
1	0,003373901
2	0,001908954
3	0,006092715

Team 2:

Satellite	Time Interval (sec)
1	0,00605192
2	0,001786836
3	0,003399051

Team 3:

Satellite	Time Interval (sec)
1	0,006775321
2	0,004257879
3	0,009741639

The first task of each team is to convert the time interval data in meters, using the known value of the speed of light given earlier:

$$c = 299792458\ m/s$$

Due to the precision of the numbers, a scientific calculator will be needed (or Microsoft Excell). Write your results at your notebook. Now, go back to the map and choose the **"Draw a Circle "** option:

Figure 9: Drawing circles on MapMaker

Starting from the position of each flag, expand the circle outwards until the distance displayed on your monitor equals the one you calculated already.

<u>Be extra careful because when you stop expanding the circle, because once you lift the cursor, the distance isn't displayed anymore.</u>

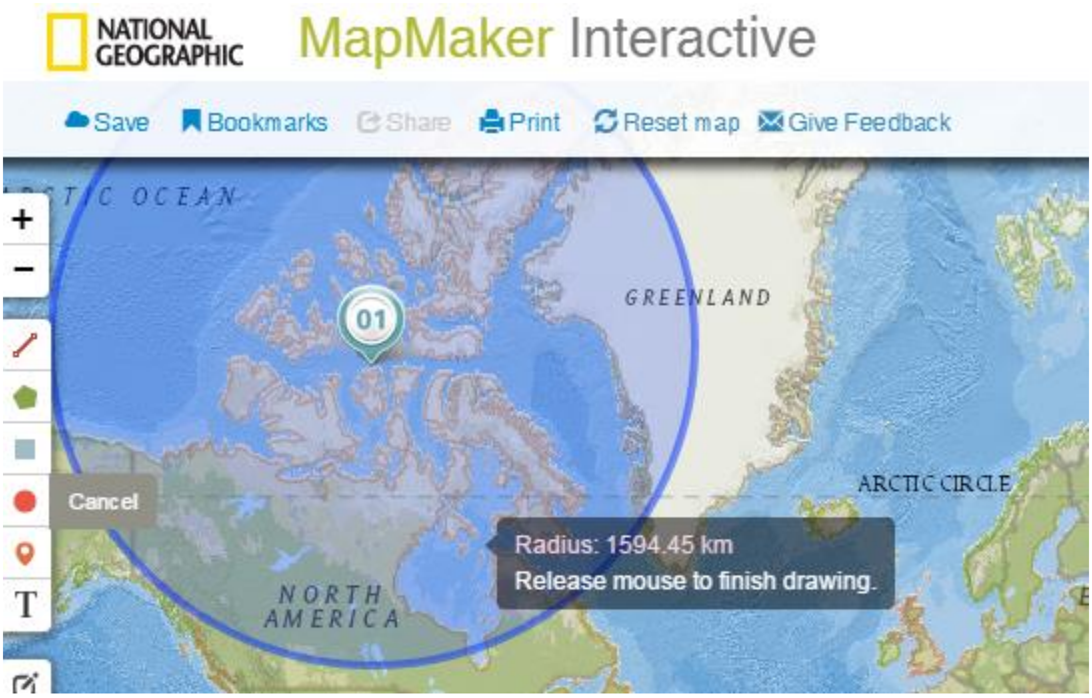

Figure 10: Draw circle of specific radius (in km)

- Do the same for each satellite.
- Find the point in which the three circles intersect and flag it with a marker of your liking (here we have chosen a flag).
- Team 1 will mark location A, team 2 will mark location B and team 3 will mark location C.

Write down the coordinates of the location you found as well as the country it belongs in. Your final result should look like this:

Figure 11: Final results and the GPS position (approximately)

You have now implemented successfully the trilateration technique!

It is advised that you take some time to get used to the controllers, and especially the expanding circle and the value of its radius. Students might have a problem in this and could need extra time to repeat their investigation. You may also choose to propose different locations for the satellites and time interval data received. If you consider that the activity takes too long, you may as well choose to work with two locations instead of three.

Discussion

Explanation based on evidence

Guide the teachers to encourage their students to provide correct explanations for the topic(s) investigated.

Describe ways and they can use to this end and give them directions how to discover them.

If you consider that you have not enough time to carry out the measurement of all three distances, you may as well choose one and proceed with it.

The desired results are the following:

	Team 1				
Satellite Nr	radius (km)	c (km/sec)	time interval (sec)		
1	1011,47	299792,458	0,003373901	Location A	
2	572,29	299792,458	0,001908954	Latitude(°)	Longitude(°)
3	1826,55	299792,458	0,006092715	49,94	6,55
Satellite Nr	Team 2				
1	1814,32	299792,458	0,00605192	Location B	
2	535,68	299792,458	0,001786836	Latitude(°)	Longitude(°)
3	1019,01	299792,458	0,003399051	53,54	-6,57
Satellite Nr	Team 3				
1	2031,19	299792,458	0,006775321	Location C	
2	1276,48	299792,458	0,004257879	Latitude(°)	Longitude(°)
3	2920,47	299792,458	0,009741639	52,5	21,07

In this section we will push one step further and calculate distances using the results we obtained so far. This is one of the major operations that can be done using the GPS and this way one can calculate the distance from a starting point to an ending point. Every team has now obtained the latitude and longitude of their position. A representative of each team writes the latitude and longitude they measured on the blackboard.

All the teams use the interactive map and flag the three locations on it.

Consider other explanations

Facilitate the student groups to evaluate their own explanations in the light of alternative explanations, particularly those reflecting scientific understanding. Illustrate examples they can use and give them instructions how to locate them.

Now, you will use the option "Draw a polyline" of the map:

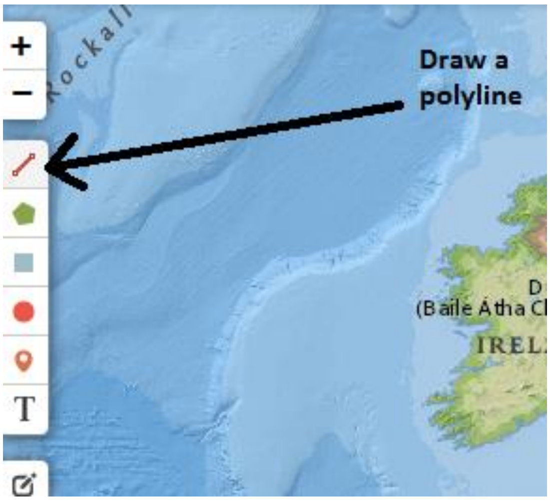

Figure 12: Drawing a polyline on MapMaker

Your task now is to use the polyline in order to measure the distances between the flagged locations on the map.

You should be extra careful concerning the distance you measure.

Save the number you obtain at the exact spot that you finished your line.

Now, use the distance calculator http://www.nhc.noaa.gov/gccalc.shtml in order to compute the actual distance between locations A, B and C. The calculator uses the same mathematical formula employed by the GPS in order to compute actual distances between locations.

Make sure that the students use as starting and ending points the exact spots that they marked when they used the trilateration method. This can be done by zooming in and out of the map until the starting and ending coordinates correspond exactly to the desired points. The results obtained using the two methods are summarized at the following table:

As we can see, the accuracy of distance determination is of the order of 8km:

Distance	Polyline (km)	Distance Calculator (km)
AB	973,6	981
AC	1050,8	1043
BC	1828,76	1830

Compare the results obtained by the two methods and discuss the deviations you observe. Consider the distances calculated by the distance calculator as the optimal.

If you zoom in the intersection points of the three circles you drew to find your location, do they overlap at exactly one point?

Can you estimate the accuracy of the method you used in your investigation?

Compare the accuracy you estimated with the accuracy of a commercial GPS. What do you observe?

Name the experimental errors you consider most important and discuss ways to eliminate them.

If you zoom in at the place where the three circles overlap, you will see that they do not coincide exactly at a point. A triangle is formed instead. If we measure the area of the triangle and create a circle of the same area, its radius can be defined as the accuracy of our method (and is of the order of a few km depending on the zoom options we used.

Higher resolution implies higher accuracy). Therefore, if we assume a 5km accuracy with the average distance between satellites and receiver being 1000km, we have an uncertainty of the order 0.5%. In comparison, a real GPS satellite is 20000km far from the surface of the earth with an error of up to 10m in determining the location of a receiver. Therefore, in a real-world example we have an uncertainty of the order 0.0000005% which is six orders of magnitude better than the method we used!

Reflection

Communicate explanation

Guide teachers to facilitate each student group to reflect on the previous experiences and to produce a report with its findings, presenting and justifying the proposed explanations to the other groups and the teacher.

Provide content which the teacher can use to help the students to get familiarized and to become efficient in scientific writing.

Students can deliver a short report of previous missions landing sites, the selected criteria for these missions and the objectives.

They can link the criteria with different spatial aspects by reviewing and discussing in parallel the criteria they used, the geophysical characteristics in the landing site they propose etc.

Computer Vision on Mars

Mars Rover movement algorithms

Discuss the difficulties you encountered during your investigation.
Did you understand the method of trilateration?

The details of the GPS functionality can be explained as follows: Each satellite emits radiowaves (which are actually light of lower frequency than the visible light) which travel at the speed of light. These radiowaves carry information concerning the exact location of the satellite and the precise time (obtained using its internal atomic clock) it was when the radiowave was emitted. The clocks of all the satellites are synchronized. You can imagine the radiowaves as concentric spheres expanding at the speed of light with the satellite in the middle. From now and on we will refer to the radiowaves from a satellite as "the signal".

Our well-known GPS device contains a radio receiver which will receive the signal from each satellite it can view. It also contains an internal clock which is synchronized with the clocks of the satellites. We consider that the radiowave has reached its destination when the spherical radiowave reaches the receiver. The signal from the closest satellites will arrive earlier while the signal from the furthest ones will arrive later.

By subtracting the time the signal started from the satellite from the time the signal arrived at the receiver, we can obtain the time interval. This time interval can be converted to distance using the fact that the speed of light is a constant of known value.

This way we can obtain the distance between each satellite and the receiver.
The distance is equal to the radius of the sphere.

The point of intersection of the four spheres is our location. This is the method of trilateration.

Follow-up activities and materials

Describe and direct the user to any follow-up activities or materials that can be used to wrap-up the hands-on activity.

These could include appropriate learning assessment and/or reminder materials (e.g., quizzes, games, other user-friendly tests), hints for further activities etc.

You can stimulate further discussion using information from the links below:
http://rachelgreenberg.tripod.com/id12.html
http://www.batteriesinaflash.com/blog/how-gps-technology-has-changed-society/

It would be optimal if the students could see a real time demonstration with a GPS if the teacher has access to one.

Take some time to answer the following quiz concerning the GPS:
http://electronics.howstuffworks.com/gadgets/travel/gps-quiz.htm

Sustainable contact

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References (if any):

Assessment (if any):